

RC Electronics

This document explores the properties of RC (Resistor/Capacitor) circuits.

Constants and Definitions:

$$k \equiv 1000 \cdot \text{ohm} \quad \text{kHz} := 1000 \cdot \text{Hz} \quad \mu\text{F} := 10^{-6} \cdot \text{farad} \quad \text{dB}(x) := 20 \cdot \log(x)$$

$$\text{MHz} := 10^6 \cdot \text{Hz}$$

Input voltage for worksheet.

$$V_{\text{in}} := 1 \cdot \text{volt}$$

Equations for Charging and Discharging RC Circuits

$$t := 0 \cdot \text{sec}, \frac{5 \cdot R \cdot C \cdot C}{500} .. 5 \cdot R \cdot C \cdot C \quad \text{Range of time for graphing, from 0 to } 5 \cdot RC$$

$$V_{\text{charge}}(t) := V_{\text{in}} \cdot \left(1 - e^{-\frac{t}{R \cdot C \cdot C}} \right) \quad \text{Equation for voltage of a charging RC circuit}$$

$$V_{\text{discharge}}(t) := V_{\text{in}} \cdot e^{-\frac{t}{R \cdot C \cdot C}} \quad \text{Equation for voltage of discharging RC circuit.}$$

Frequency Range for graphing filter response:

$$\text{Low Frequency Limit} \quad f_{\text{low}} := 0.01 \cdot \text{Hz}$$

$$\text{High Frequency Limit} \quad f_{\text{high}} := 10 \cdot \text{MHz}$$

Frequency Index

$$f := \log(f_{\text{low}} \cdot \text{Hz}^{-1}), \log(f_{\text{low}} \cdot \text{Hz}^{-1}) + \left[\frac{\log\left(\frac{f_{\text{high}}}{f_{\text{low}}}\right)}{500} \right] .. \log(f_{\text{high}} \cdot \text{Hz}^{-1})$$

$$g(f) := 10^f \cdot \text{Hz}$$

Equations for RC Filters:

$$A_{HP}(f) := \frac{(2 \cdot \pi \cdot g(f)) \cdot R_F \cdot C_F}{\sqrt{(2 \cdot \pi \cdot g(f))^2 \cdot R_F^2 \cdot C_F^2 + 1}}$$

Amplitude of high pass filter

$$A_{LP}(f) := \frac{1}{\sqrt{(2 \cdot \pi \cdot g(f))^2 \cdot R_F^2 \cdot C_F^2 + 1}}$$

Amplitude of low pass filter

$$\phi_{HP}(f) := \text{atan} \left[\frac{1}{(2 \cdot \pi \cdot g(f)) \cdot R_F \cdot C_F} \right]$$

Phase of high pass filter

$$\phi_{LP}(f) := \left[-\text{atan} \left[\frac{-1}{(2 \cdot \pi \cdot g(f)) \cdot R_F \cdot C_F} \right] - \frac{\pi}{2} \right]$$

Phase of low pass filter

Equations for scope output

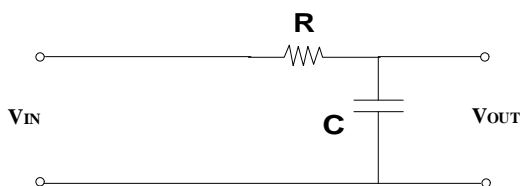
$$f_{\text{signal}} := \log(\text{frequency} \cdot \text{sec}) \quad t_{\text{scope}} := 0 \cdot \text{sec}, \frac{5}{\text{frequency} \cdot 500} \dots \frac{5}{\text{frequency}}$$

$$V_{\text{signal}}(t) := \sin((2 \cdot \pi \cdot \text{frequency}) \cdot t)$$

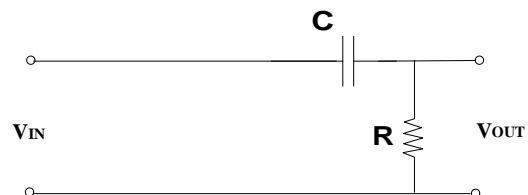
$$V_{LP}(t) := A_{LP}(f_{\text{signal}}) \cdot \sin \left[(2 \cdot \pi \cdot \text{frequency}) \cdot t + \phi_{LP}(f_{\text{signal}}) \right]$$

$$V_{HP}(t) := A_{HP}(f_{\text{signal}}) \cdot \sin \left[(2 \cdot \pi \cdot \text{frequency}) \cdot t + \phi_{HP}(f_{\text{signal}}) \right]$$

Low Pass Filter



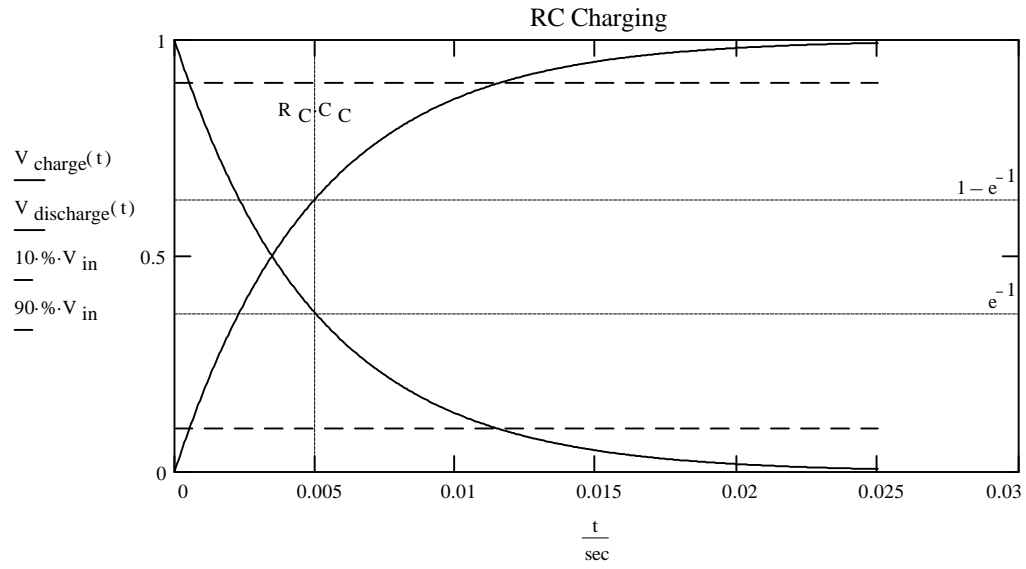
High Pass Filter



Charging and Discharging of RC Circuit

Resistance of Circuit $R_C \equiv 5 \cdot k$

Capacitance of Circuit $C_C \equiv 1 \cdot \mu F$



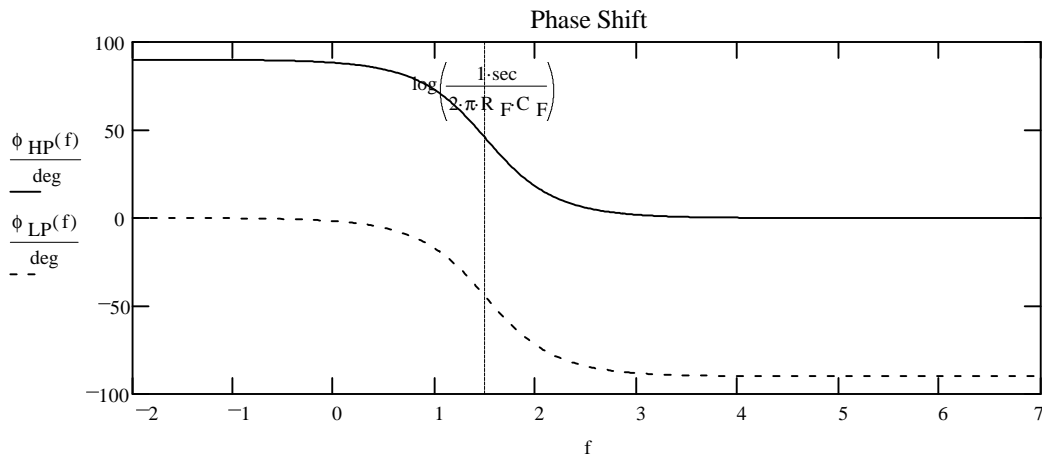
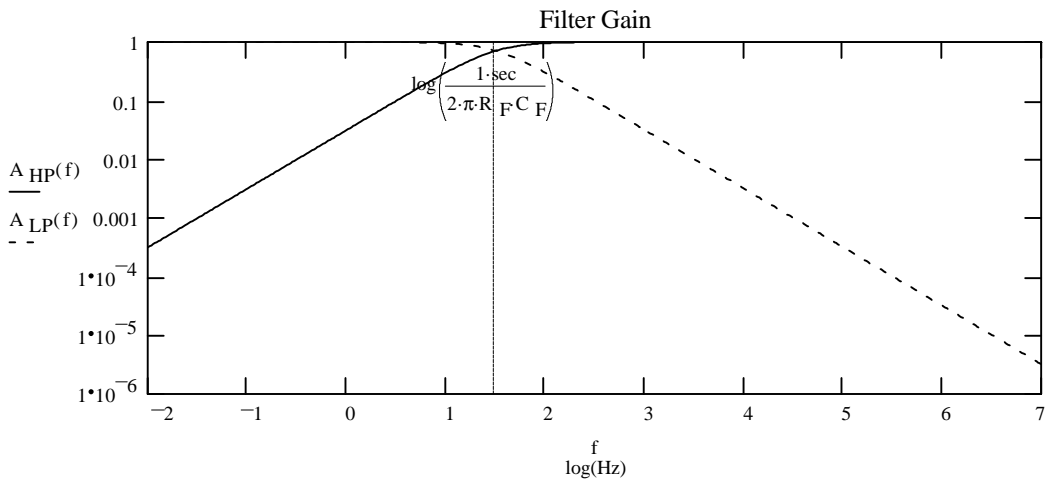
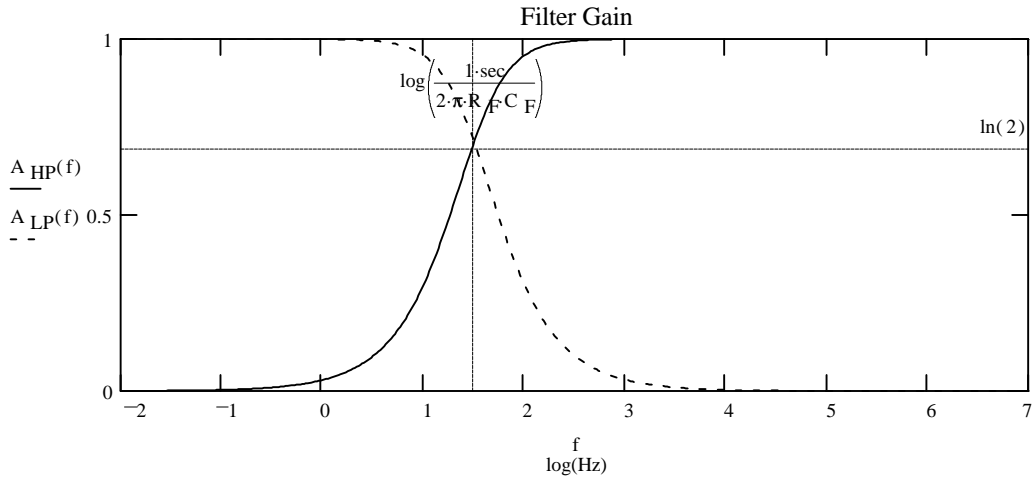
Frequency and Phase Response of High Pass and Low Pass RC Filter

$$R_F \equiv 5 \cdot k$$

$$C_F \equiv 1 \cdot \mu F$$

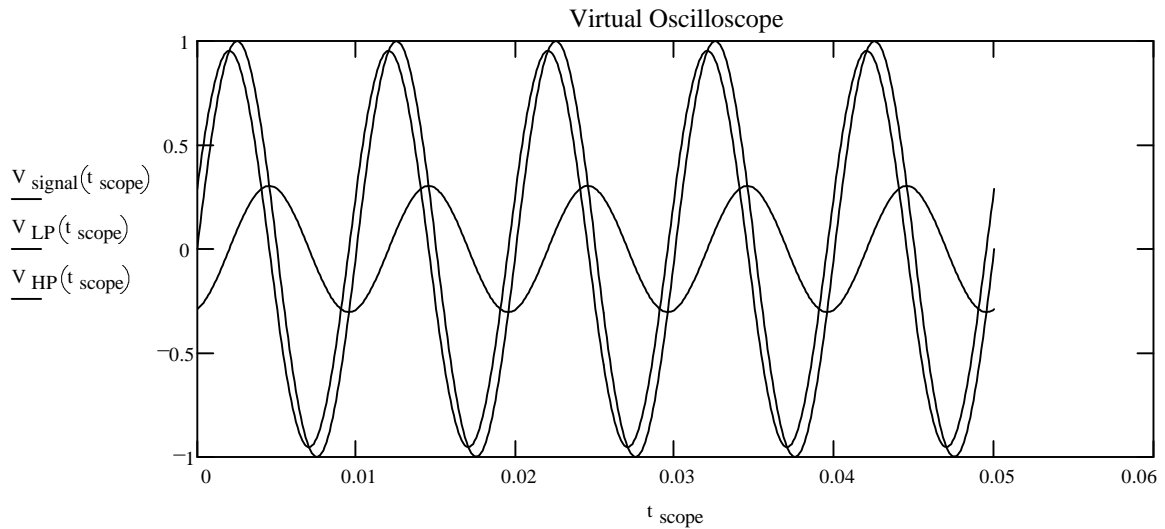
$$R_F \cdot C_F = 0.005 \cdot \text{sec}$$

$$\frac{1}{2 \cdot \pi \cdot R_F \cdot C_F} = 31.831 \cdot \text{sec}^{-1}$$



Waveform for RC Filters (Virtual Oscilloscope)

Input Frequency frequency = 100 · Hz $\frac{1}{2 \cdot \pi \cdot R \cdot C_F} = 31.831 \cdot \text{Hz}$



Circuit Gain:

Low Pass $A_{LP}(f_{\text{signal}}) = 0.303$

High Pass $A_{HP}(f_{\text{signal}}) = 0.953$

Circuit Gain (in decibals):

Low Pass $\text{dB}(A_{LP}(f_{\text{signal}})) = -10.362$

High Pass $\text{dB}(A_{HP}(f_{\text{signal}})) = -0.419$

Circuit Phase Shift:

Low Pass $\phi_{LP}(f_{\text{signal}}) = -0.402 \cdot \pi \cdot \text{rad}$ $\phi_{LP}(f_{\text{signal}}) = -72.343 \cdot \text{deg}$

High Pass $\phi_{HP}(f_{\text{signal}}) = 0.098 \cdot \pi \cdot \text{rad}$ $\phi_{HP}(f_{\text{signal}}) = 17.657 \cdot \text{deg}$

Questions:

To answer the questions, change the values in the yellow fields.

To read coordinates from a plot: Click on the plot, then choose Trace from the X-Y Plot menu. Click and drag the mouse over the points whose coordinates you want to see. Press Alt+F4 to dismiss the dialog box.

1. Using a 1 μF capacitor and 1k, 10k, 100k, 500k resistors, look at the RC charging and discharging graph. Repeat using a 10k resistor and 1, 0.1, 0.001, 0.0001 μF capacitors. Make a table that shows the value of the capacitor, resistor, RC time constant, half life, and time for 90% charging. What is the significance of the RC time constant?
2. How do the RC Charging plots relate to first order kinetics? Look at your general chemistry textbook and find the equation for a first order reaction. Compare these equations. What variable in the kinetics equation is equivalent to the RC time constant?
3. Using a 1 μF capacitor and 1k, 10k, 100k, 500k resistors, what frequencies are blocked and transmitted by the low and high pass filters?
4. Using a 10k resistor and 1, 0.1, 0.001, 0.0001 μF capacitors, what frequencies are blocked and transmitted by the low and high pass filters?
5. How does the phase of the signal change in a high and low pass filter for a signal that is transmitted and a signal that is attenuated?
6. Why are logarithmic scales used?
7. Use the page for the "Waveform for RC Filters" to study how a sine wave looks after passing through a high pass or a low pass filter. Change the input frequency and observe the waveforms. Pay close attention to the amplitude and phase of the waves.

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