The Fourier Transform, Part III:

The Fourier transform is a mathematical method to describe a continuous function as a series of sine and cosine functions. This document shows how a combination of real and imaginary spectra (cosine and sine) describe the frequency and phase of a signal.

In this document the data sets are treated as arrays instead of as functions.

(Note: by convention the cosine component is called the real and the sine component is called the imaginary.)

Sampling parameters:

Number of data points $N := 512$

Total time the signal is acquired $acquire := 1 \cdot sec$

Indexes used for timing:

$i := 0, 1 .. N - 1$ $t_i := \frac{i}{N} \cdot acquire$ $j := 0, 1 .. \frac{N}{2} - 1$ $frequency_j := \frac{j}{acquire}$

Signal

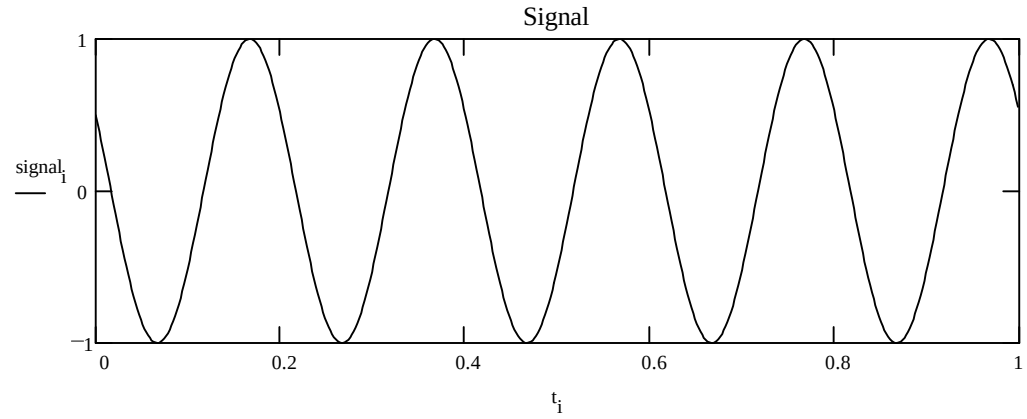
Frequency $\nu := 5 \cdot Hz$ $\omega := 2 \cdot \pi \cdot \nu$

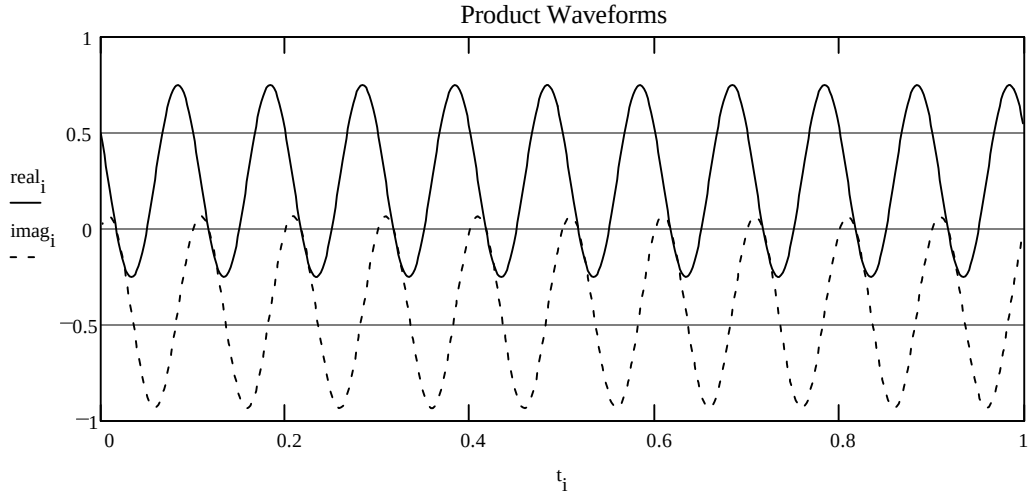
Amplitude: $A := 1$

$signal_i := A \cdot \cos\left(t_i \cdot \omega + \phi\right)$

	Real	Imaginary
Test Wave	$test_{real_i} := \cos\left(t_i \cdot \omega\right)$	$test_{imag_i} := \sin\left(t_i \cdot \omega\right)$
Product Wave	$real_i := test_{real_i} \cdot signal_i$	$imag_i := test_{imag_i} \cdot signal_i$
Integrate	$signal_{real} := \sum_{i=0}^{N-1} \frac{real_i}{0.5 \cdot N}$	$signal_{imag} := \sum_{i=0}^{N-1} \frac{imag_i}{0.5 \cdot N}$

$F := fft(signal)$





Phase: $\phi \equiv 60 \cdot deg$

Integration $signal_{real} = 0.5$

$signal_{imag} = -0.866$

Questions.

1. Change the signal phase from 0 degrees to 360 degrees in 30 degree steps. For each step, observe the following:
   a. What happens to the signal waveform?
   b. What happens to the real and imaginary product waveforms?
   c. Record the integration results for the real and imaginary signal.

2. Graph the real and imaginary signal vs the phase angle, what does this graph represent?

3. Compare the product waveforms and the integration resuts when:
   a. The signal phase is + 30 and -30 degrees.
   b. The signal phase is +60 and -60 degrees.
   c. The signal phase is +90 and -90 degrees.

This document was developed by:
Scott E. Van Bramer, Department of Chemistry
Widener University, Chester, PA 19013
svanbram@science.widener.edu
http://science.widener.edu/~svanbram