Regression Analysis Worksheet

S.E. Van Bramer
8/30/96

This worksheet preforms regression analysis using matrix manipulations. 1st (linear), 2nd (quadratic), 3rd (cubic), 4th (quartic), and 5th (?) order regressions are all preformed. The equation given is the "best fit" for the given data with that order equation. Use extreme caution, especially with higher order fits, when extrapolating these equations beyond the range of the data set.

Enter Data Set

Number of data points: $N := 14$

$i := 0, 1 .. (N - 1)$

X-Data Set

$$x := \begin{bmatrix} 1 & 150 \\ 1 & 140 \\ 1 & 130 \\ 1 & 120 \\ 1 & 110 \\ 1 & 100 \\ 1 & 90 \\ 1 & 80 \\ 1 & 70 \\ 1 & 60 \\ 1 & 50 \\ 1 & 40 \\ 1 & 30 \\ 1 & 20 \\ 1 & 10 \end{bmatrix}$$

Y-Data Set

$$y := \begin{bmatrix} 97 \\ 85 \\ 75 \\ 65 \\ 56 \\ 48 \\ 42 \\ 35 \\ 30 \\ 25 \\ 20 \\ 16 \\ 12 \\ 7 \\ 5 \end{bmatrix}$$

Linear Regression y = m*x + b

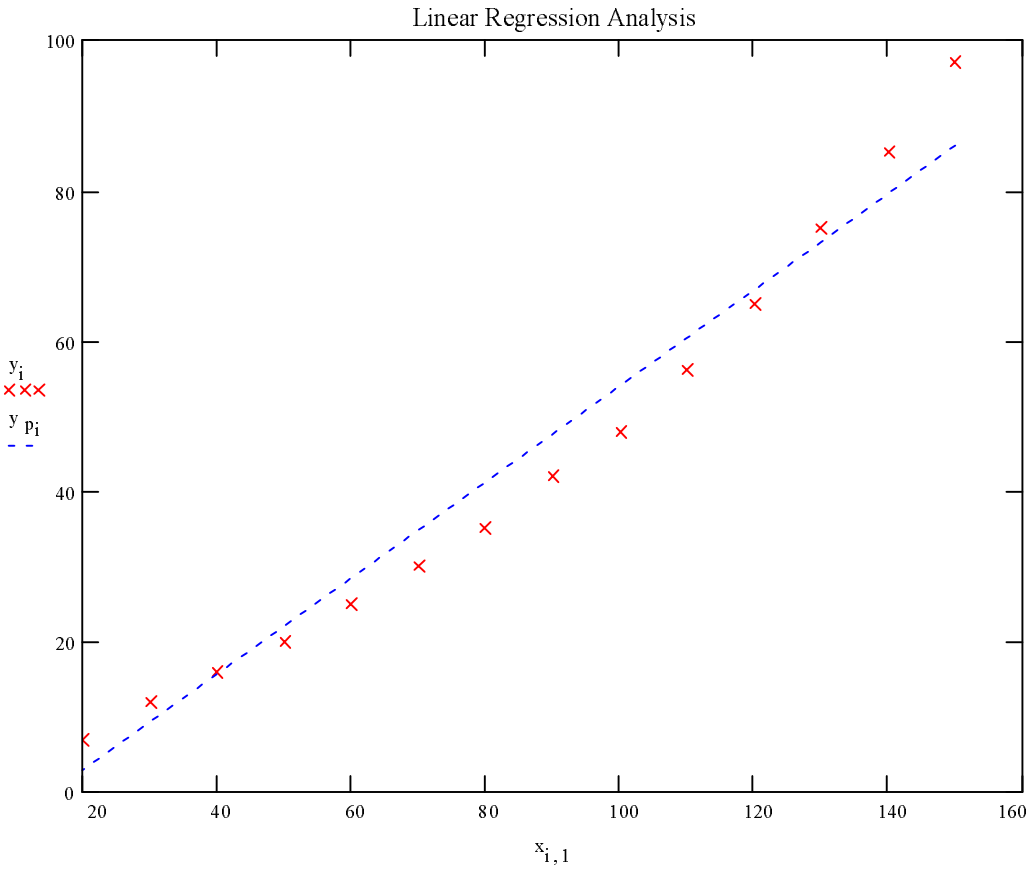
Regression Analysis

$$z := \left(x^T \cdot x\right)^{-1} \cdot \left(x^T \cdot y\right)$$

Slope $m := z_1$

Intercept $b := z_0$

Predicted $y_p := m \cdot x^{\langle 1 \rangle} + b$



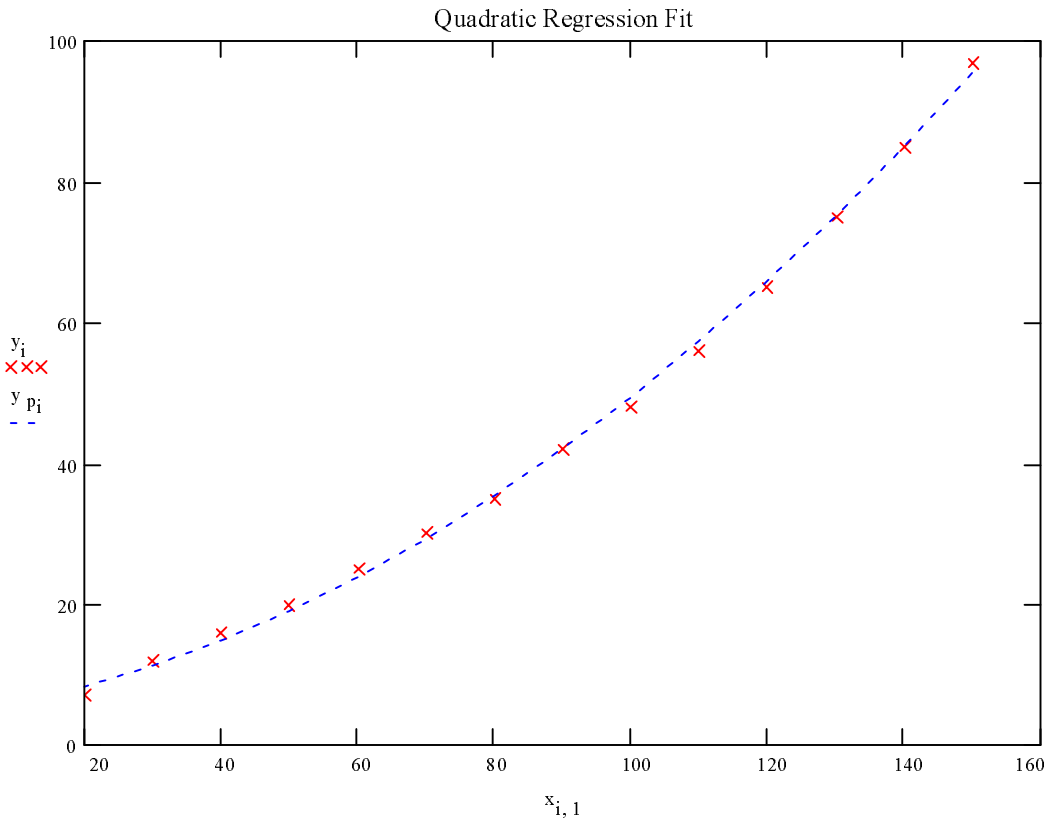
Quadratic Regression ($y = m_2 x^2 + m_1 x + b$)

$$x^{\langle 2 \rangle} := \overrightarrow{\left(x^{\langle 1 \rangle}\right)^2}$$

Regression Analysis

$$z := \left(x^T \cdot x\right)^{-1} \cdot \left(x^T \cdot y\right)$$

Slope	$m_1 := z_1$	$m_1 = 0.137$
	$m_2 := z_2$	$m_2 = 0.003$
Intercept	$b := z_0$	$b = 4.295$
Predicted	$y_p := m_2 \cdot x^{\langle 2 \rangle} + m_1 \cdot \left(x^{\langle 1 \rangle}\right) + b$	



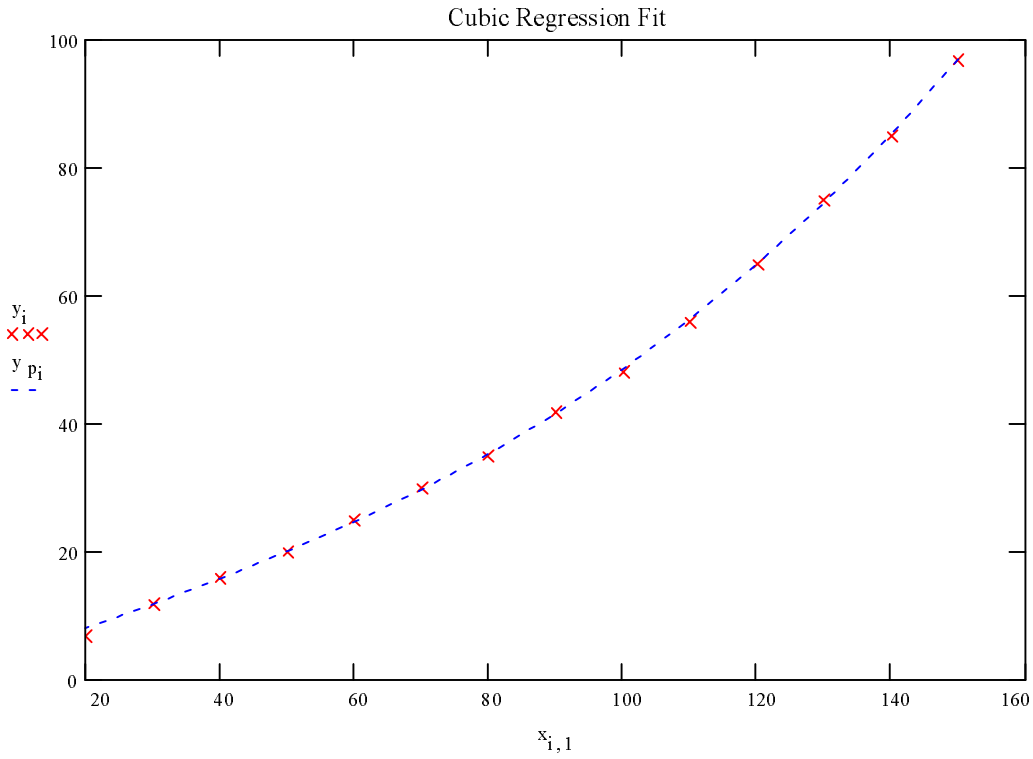
Cubic Regression ($y = m_3*x^3 + m_2*x^2 + m_{31}*x + b$)

$$x^{\langle 3 \rangle} := \overrightarrow{\left(x^{\langle 1 \rangle}\right)^3}$$

Regression Analysis

$$z := \left(x^T \cdot x\right)^{-1} \cdot \left(x^T \cdot y\right)$$

Slope	$m_1 := z_1$	$m_1 = 0.367$
	$m_2 := z_2$	$m_2 = -3.448 \cdot 10^{-4}$
	$m_3 := z_3$	$m_3 = 1.45 \cdot 10^{-5}$
Intercept	$b := z_0$	$b = 0.746$
Predicted	$y_p := m_3 \cdot x^{\langle 3 \rangle} + m_2 \cdot x^{\langle 2 \rangle} + m_1 \cdot x^{\langle 1 \rangle} + b$	



Quartic Regression ($y = m_4*x^4 + m_3*x^3 + m_2*x^2 + m_1*x + b$)

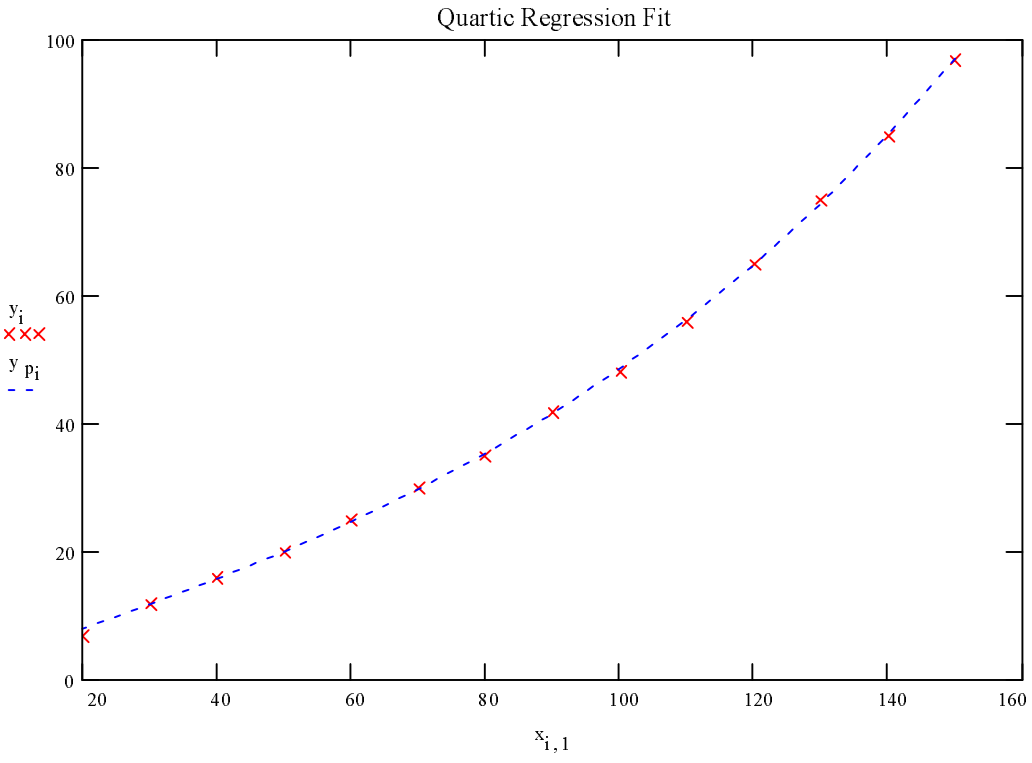
$$x^{\langle 4 \rangle} := \overrightarrow{\left(x^{\langle 1 \rangle}\right)^4}$$

Regression Analysis

$$z := \left(x^T \cdot x\right)^{-1} \cdot \left(x^T \cdot y\right)$$

Slope	$m_1 := z_1$	$m_1 = 0.334$
	$m_2 := z_2$	$m_2 = 5.33 \cdot 10^{-4}$
	$m_3 := z_3$	$m_3 = 6.154 \cdot 10^{-6}$
	$m_4 := z_4$	$m_4 = 2.607 \cdot 10^{-8}$
Intercept	$b := z_0$	$b = 1.092$

Predicted $\quad y_p := m_4 \cdot x^{\langle 4 \rangle} + m_3 \cdot x^{\langle 3 \rangle} + m_2 \cdot x^{\langle 2 \rangle} + m_1 \cdot x^{\langle 1 \rangle} + b$



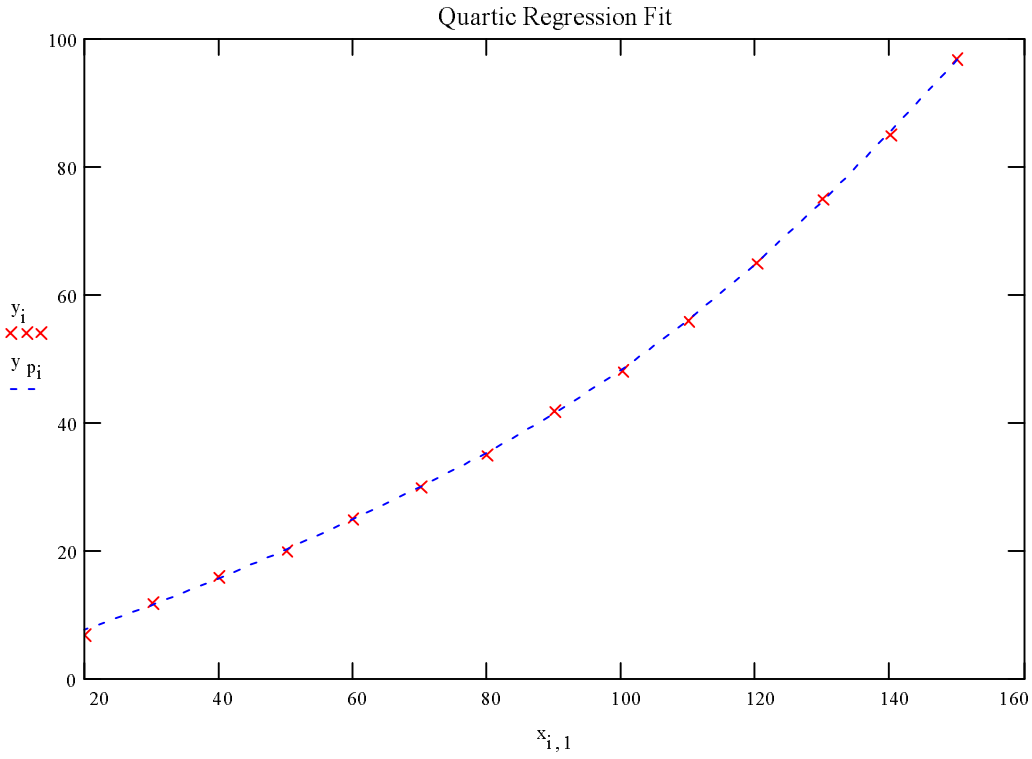
5th Order Regression ($y = m_5*x^5 + m_4*x^4 + m_3*x^3 + m_2*x^2 + m_1*x + b$)

$$x^{\langle 5\rangle} := \overrightarrow{\left(x^{\langle 1\rangle}\right)^5}$$

Regression Analysis

$$z := \left(x^T \cdot x\right)^{-1} \cdot \left(x^T \cdot y\right)$$

Slope	$m_1 := z_1$	$m_1 = 0.065$
	$m_2 := z_2$	$m_2 = 0.011$
	$m_3 := z_3$	$m_3 = -1.57 \cdot 10^{-4}$
	$m_4 := z_4$	$m_4 = 1.152 \cdot 10^{-6}$
	$m_5 := z_5$	$m_5 = -2.815 \cdot 10^{-9}$
Intercept	$b := z_0$	$b = 3.171$
Predicted	$y_p := m_5 \cdot x^{\langle 5\rangle} + m_4 \cdot x^{\langle 4\rangle} + m_3 \cdot x^{\langle 3\rangle} + m_2 \cdot x^{\langle 2\rangle} + m_1 \cdot x^{\langle 1\rangle} + b$	



This document was developed by:
Scott E. Van Bramer
Department of Chemistry
Widener University
Chester, PA 19013
svanbram@science.widener.edu
http://science.widener.edu