Linear Regression Analysis

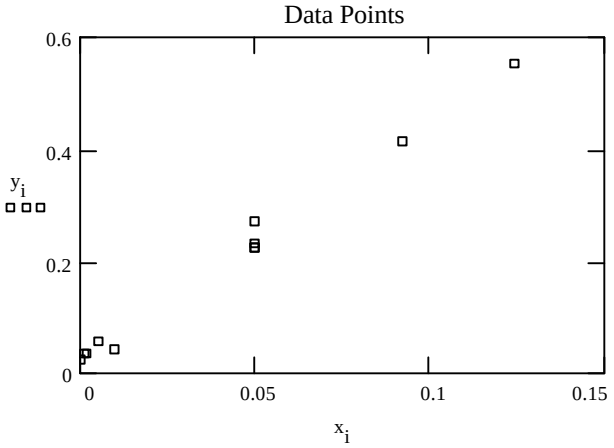
This document preforms a linear regression analysis on the x, y data set. In addition to the slope and intercept, the regression, slope, and intercept are calculated. At the end of this document is a section for applying the regression analysis to unknowns.

Data Set

$N := 11$

$i := 1, 2 .. N$

$x_i :=$	$y_i :=$	$x_i \cdot y_i$	$(y_i)^2$	$(x_i)^2$
0	0.02599	0	$6.755 \cdot 10^{-4}$	0
0.001	0.03544	$3.544 \cdot 10^{-5}$	0.001	$1 \cdot 10^{-6}$
0.002	0.03447	$6.894 \cdot 10^{-5}$	0.001	$4 \cdot 10^{-6}$
0.005	0.05885	$2.943 \cdot 10^{-4}$	0.003	$2.5 \cdot 10^{-5}$
0.01	0.04349	$4.349 \cdot 10^{-4}$	0.002	$1 \cdot 10^{-4}$
0.05	0.23143	0.012	0.054	0.003
0.05	0.22492	0.011	0.051	0.003
0.05	0.22656	0.011	0.051	0.003
0.05	0.27000	0.014	0.073	0.003
0.092	0.41289	0.038	0.17	0.008
0.124	0.55200	0.068	0.305	0.015



Regression Analysis:

$$x_{avg} := \sum_i \frac{x_i}{N} \qquad x_{avg} = 0.039$$

$$y_{avg} := \sum_i \frac{y_i}{N} \qquad y_{avg} = 0.192$$

$$s_{xy} := \sum_i \left(x_i \cdot y_i\right) - \left[\frac{\left(\sum_i x_i\right) \cdot \left(\sum_i y_i\right)}{N}\right] \qquad s_{xy} = 0.071$$

$$s_{yy} := \sum_i \left(y_i\right)^2 - \left[\frac{\left(\sum_i y_i\right) \cdot \left(\sum_i y_i\right)}{N}\right] \qquad s_{yy} = 0.305$$

$$s_{xx} := \sum_i \left(x_i\right)^2 - \left[\frac{\left(\sum_i x_i\right) \cdot \left(\sum_i x_i\right)}{N}\right] \qquad s_{xx} = 0.017$$

Calculation of line:

Slope:

$$m := \frac{s_{xy}}{s_{xx}} \qquad m = 4.24$$

Intercept:

$$b := y_{avg} - \left(m \cdot x_{avg}\right) \qquad b = 0.025$$

$$y_{calc_i} := m \cdot \left(x_i\right) + b$$

Uncertainty Calculations:

In the regression

$$s_r := \sqrt{\frac{s_{yy} - \left(m^2 \cdot s_{xx}\right)}{N - 2}} \qquad s_r = 0.015$$

In the slope

$$s_m := \sqrt{\frac{s_r^{\,2}}{s_{xx}}} \qquad s_m = 0.119$$

In the intercept

$$s_b := s_r \cdot \sqrt{\frac{\sum_i \left(x_i\right)^2}{N \cdot \sum_i \left(x_i\right)^2 - \left(\sum_i x_i\right)^2}} \qquad s_b = 0.007$$

Analyisis of an Unknown:

replicates $M := 1$ $j := 1, 2 .. M$

$signal_j := 0.21535$ $signal_{avg} := \sum_j \frac{signal_j}{M}$

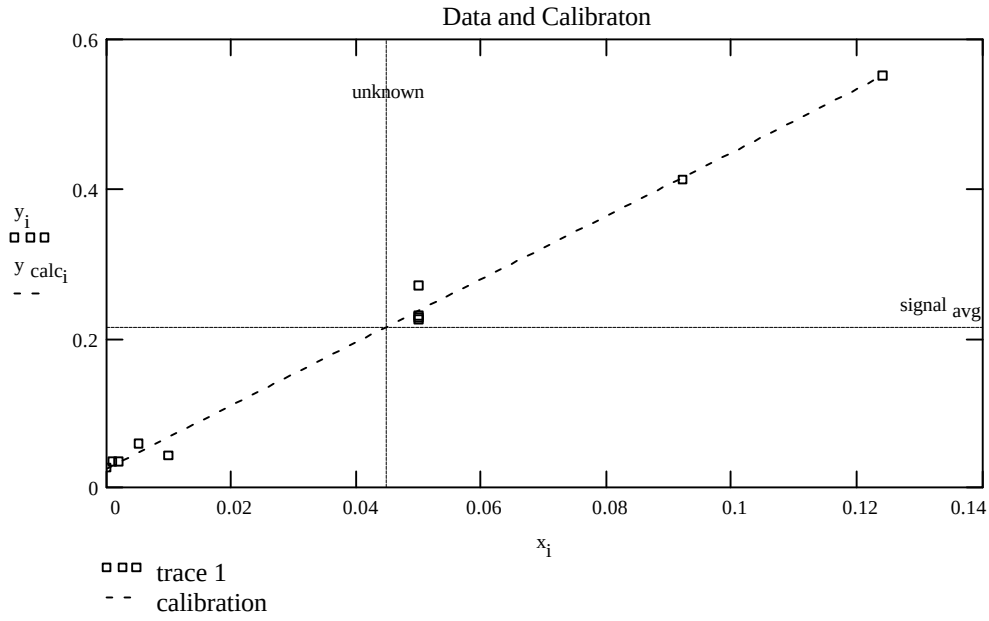
Calculation of unknown

$$unknown := \frac{signal_{avg} - b}{m} \qquad unknown = 0.045$$

Calculation of uncertainty in unknown

$$s_{unknown} := \left(\frac{s_r}{m}\right) \cdot \sqrt{\frac{1}{M} + \frac{1}{N} + \frac{\left(signal_{avg} - y_{avg}\right)^2}{m^2 \cdot s_{xx}}} \qquad s_{unknown} = 0.004$$

$$RSD := \frac{s_{unknown}}{unknown} \qquad RSD = 8.498 \cdot \%$$



This document prepared by:
S.E. Van Bramer
Department of Chemistry
Widener University
Chester, PA 19013
svanbram@science.widener.edu
http://science.widener.edu/~svanbram