

Significant Digits Examples

S.E. Van Bramer,

Addition and Subtraction

Round to:

$$\begin{array}{r} 12.4 \dots = 28.18000 \\ + 15.78 \end{array}$$

28.2

$$\begin{array}{r} 167.34 \dots = 1114.64000 \\ + 947.3 \end{array}$$

1114.6

$$\begin{array}{r} 495.367 \dots = 458.12700 \\ + -37.24 \end{array}$$

458.13

$$\begin{array}{r} 1.73485 \dots = 1132.04685 \\ + 145.182 \dots \\ + 985.13 \end{array}$$

1132.05

$$\begin{array}{r} 421 \dots = 437.48650 \\ + 16.4865 \end{array}$$

437

$$\begin{array}{r} 231 \dots = 248.54000 \\ + 17.54 \end{array}$$

249

$$\begin{array}{r} 232 \dots = 249.56000 \\ + 17.56 \end{array}$$

250.

Multiplication and Division:

$$2.3 = 6.00000$$

6

$$45.3 \cdot 27.4 = 1241.22000$$

$1.24 \cdot 10^3$

$$1.5487 \cdot 12.2 = 18.89414$$

18.9

$$\frac{56.7}{3.4} = 16.67647$$

17

$$(1.465 \cdot 10^4) \cdot (4.68 \cdot 10^{-2}) = 685.62000$$

686

Place two sticks end to end. The length of the first stick is measured with a meter stick as 59.3 cm. The second stick is measured with a micrometer as 5.4895 cm. What is the total length?

$$\begin{array}{r} 59.3 \dots = 64.7895 \\ + 5.4895 \end{array} \quad 64.8$$

Change length of second stick to 5.1594 cm. What is the total length?

$$\begin{array}{r} 59.3 \dots = 64.45950000 \\ + 5.1594 \end{array} \quad 64.5 \quad \text{Since the digit after the 4 in the tenth's place is a 5, round the 4 up (5).}$$

Measure the length of the first stick with a better ruler, as 59.394 cm. Now what is the length of the two sticks?

$$\begin{array}{r} 59.394 \dots = 64.55350000 \\ + 5.1594 \end{array} \quad 64.554 \quad \text{Since the digit after the 3 in the thousandth's place is a 5, round 3 up (4).}$$

You are given a rectangular piece of metal. Using different devices you measure the following dimensions: Length 25.125 inches, Width 5.3 cm, Thickness 1.3458 mm. What is the volume of this piece?

$$\text{volume} = \text{length} \cdot \text{width} \cdot \text{height}$$

Given in the problem:

$$\text{length} := 25.125 \cdot \text{in}$$

$$\text{width} := 5.3 \cdot \text{cm}$$

$$\text{height} := 1.3458 \cdot \text{mm}$$

All measurements need to be in the same units. I'll pick cm.

$$25.125 \cdot \text{in} \cdot \left(\frac{2.54 \cdot \text{cm}}{1 \cdot \text{in}} \right) = 63.81750 \text{ cm}$$

$$5.3 \cdot \text{cm} = 5.30000 \text{ cm}$$

$$1.3458 \cdot \text{mm} \cdot \left(\frac{10^{-3} \cdot \text{m}}{\text{mm}} \right) \cdot \left(\frac{1 \cdot \text{cm}}{10^{-2} \cdot \text{m}} \right) = 0.13458 \text{ cm}$$

Calculate volume

$$\text{volume} := \text{length} \cdot \text{width} \cdot \text{height}$$

$$\text{volume} = 45.51936 \text{ cm}^3$$

$$46 \cdot \text{cm}^3$$

Problem Solving. While driving across South Dakota, I looked down at the odometer in my truck and started a stop watch at the same time. The odometer read: 129356.4. Some time latter I stoped the watch and looked at the odometer again. The odometer read: 129469.3 and the stopwatch read 94 min, 15.24 sec. What was my speed in miles per hour, kilometers per hour and meters per second?

To find velocity we need distance (from the odometer) and time (from the stopwatch). First the distance.

$$\text{start} := 129356.4 \cdot \text{mi}$$

$$\text{stop} := 129469.3 \cdot \text{mi}$$

$$\text{distance} := \text{stop} - \text{start}$$

$$\text{distance} = 112.90000 \text{ mi}$$

Look at the start and stop readings to determine how many significant digits the answer should have at this point. The start is in tenths and the stop is in tenths. The distance should be the LEAST precise of the two. They are the same, so this is easy. The value should be in tenth's at this point. The following digit (0 in the hundredth's place) is less than 5, so round the 9 to 9.

$$\text{distance} = 112.90000 \text{ mi}$$

Now calculate the time:

$$\text{time} := 94 \cdot \text{min} \cdot \left(\frac{60 \cdot \text{sec}}{1 \cdot \text{min}} \right) + 15.24 \cdot \text{sec}$$

$$\text{time} = 5655.24000 \text{ sec}$$

But how do we figure the significant figures? Answer this question by thinking about where the uncertainty is in the measurement. Remember, this is what significant figures represent. Is the uncertainty in the 94 min? Is it in the 15 sec? Or is it the two tenth's of a second? Or is it the 4 hundredth's of a second? So this answer is:

$$\text{time} = 5655.24 \text{ sec}$$

To calculate miles per hour, the time must be converted into hours.

$$\text{time} \cdot \left(\frac{60 \cdot \text{sec}}{\text{min}} \right) \cdot \left(\frac{60 \cdot \text{min}}{\text{hr}} \right) = 1.57090000 \text{ hr}$$

Since the conversion factors are "exact" numbers, the solution should have the same number of significant figures as the time in seconds (the rule for multiplication and division). So this is written as:

$$\text{time} = 1.57090 \text{ hr}$$

$$\text{velocity} := \frac{\text{distance}}{\text{time}}$$

$$\text{velocity} = 71.86962888 \frac{\text{mi}}{\text{hr}} \quad \text{or} \quad \frac{112.9 \cdot \text{mi}}{1.57090 \cdot \text{hr}} = 71.86962888 \frac{\text{mi}}{\text{hr}}$$

From the rules for significant figures with multiplication and division. 112.9 has 4 significant digits, 1.56090 has six significant digits. 4 is the "least" number of significant digits, so the answer should have 4 significant digits. This means rounding to the 6 in the hundredth's place. Because the following digit is a 9, which is more than 5, the 6 rounds up to a 7. The answer is written as:

$$71.87 \cdot \frac{\text{mi}}{\text{hr}}$$

Next convert this to kilometers per hour:

The necessary conversion factor (to do this in one step) is:

$$1 \cdot \text{mi} = 1.609344 \text{ km}$$

Note: It is important to not have the conversion factor limit the uncertainty in your result. Always try and find a value for the conversion factor that has as many or more significant digits than your experimental measurement. You may look for conversion factors:

In the back of your textbook

In the CRC Handbook of Chemistry and Physics

At the NIST website (<http://physics.nist.gov/Pubs/SP811/contents.full.html>)

The conversion is:

$$71.87 \cdot \frac{\text{mi}}{\text{hr}} \cdot \left(\frac{1.609344 \cdot \text{km}}{1 \cdot \text{mi}} \right) = 115.66355 \frac{\text{km}}{\text{hr}} \quad 115.7 \cdot \frac{\text{km}}{\text{hr}}$$

Finally meters per second:

$$115.7 \cdot \frac{\text{km}}{\text{hr}} \cdot \left(\frac{10^3 \cdot \text{m}}{1 \cdot \text{km}} \right) \cdot \left(\frac{1 \cdot \text{hr}}{60 \cdot \text{min}} \right) \cdot \left(\frac{1 \cdot \text{min}}{60 \cdot \text{sec}} \right) = 32.13888889 \text{ m sec}^{-1} \quad 32.14 \cdot \frac{\text{m}}{\text{sec}}$$

Notice, if all conversions are carried out at once the calculated result is:

$$\text{velocity} = 32.12859889 \text{ m sec}^{-1}$$

Which rounds to:

$$32.13 \cdot \frac{\text{m}}{\text{sec}}$$

Since the last digit is uncertain, it is reasonable that the result's may vary by one at this position. Either answer is acceptable.