

Solutions to:

Unit Conversions and Significant Figures Problem Set

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1. Conversions: Express the following in the units asked for:

a. Speed of light, $3.00 \times 10^8 \text{ m s}^{-1}$, in miles per hour and meters/sec.

One possible set of conversion factors is:

$$3.00 \cdot 10^8 \cdot \frac{\text{m}}{\text{sec}} \cdot \left(\frac{100 \cdot \text{cm}}{1 \cdot \text{m}} \right) \cdot \left(\frac{1 \cdot \text{in}}{2.54 \cdot \text{cm}} \right) \cdot \left(\frac{1 \cdot \text{ft}}{12 \cdot \text{in}} \right) \cdot \left(\frac{1 \cdot \text{mi}}{5280 \cdot \text{ft}} \right) \cdot \left(\frac{60 \cdot \text{sec}}{1 \cdot \text{min}} \right) \cdot \left(\frac{60 \cdot \text{min}}{1 \cdot \text{hr}} \right) = 6.71080888 \times 10^8 \frac{\text{mi}}{\text{hr}}$$

Double check with mathcad:

$$3.00 \cdot 10^8 \cdot \frac{\text{m}}{\text{sec}} = 6.71080888 \times 10^8 \frac{\text{mi}}{\text{hr}} \quad \text{Remember, Mathcad does not work the significant figures. How many should there be in the answer?}$$

b. Speed of sound, 740 mph, in kilometers per hour and meters/sec.

Convert using Ratio's:

$$740 \cdot \frac{\text{mi}}{\text{hr}} \cdot \left(\frac{1.609 \cdot \text{km}}{\text{mi}} \right) = 1.19066 \times 10^3 \frac{\text{km}}{\text{hr}}$$

$$740 \cdot \frac{\text{mi}}{\text{hr}} \cdot \left(\frac{1.609 \cdot \text{km}}{\text{mi}} \right) \cdot \left(\frac{10^3 \cdot \text{m}}{1 \cdot \text{km}} \right) \cdot \left(\frac{1 \cdot \text{hr}}{60^2 \cdot \text{sec}} \right) = 330.73888889 \text{ m} \cdot \text{s}^{-1}$$

c. Length of a C-C bond in diamond, 1.54452 (angstroms), in cm, m, and inches (watch significant figures on this one)

$$\overset{\text{Å}}{\text{A}} := 10^{-10} \cdot \text{m}$$

$$1.54452 \cdot \text{A} \cdot \left(\frac{10^{-10} \cdot \text{m}}{1 \cdot \text{A}} \right) \cdot \left(\frac{100 \cdot \text{cm}}{1 \cdot \text{m}} \right) = 1.54452 \times 10^{-8} \text{ cm} \quad \text{The conversion factors are "exact" numbers, so they don't count for determining the number of significant digits in the answer.}$$

$$1.54452 \cdot \text{A} \cdot \left(\frac{10^{-10} \cdot \text{m}}{1 \cdot \text{A}} \right) = 1.54452 \times 10^{-10} \text{ m}$$

$$1.54452 \cdot \text{A} \cdot \left(\frac{10^{-10} \cdot \text{m}}{1 \cdot \text{A}} \right) \cdot \left(\frac{100 \cdot \text{cm}}{1 \cdot \text{m}} \right) \cdot \left(\frac{1 \cdot \text{in}}{2.54 \cdot \text{cm}} \right) = 6.0807874 \times 10^{-9} \text{ in}$$

∴, I set Mathcad
in'splay extra
_s in the answer.
How many
significant figures
should there be?
How should you
round this?

d. Temperature

i. LN2, -195 C, in F and K

$$\text{LN2}_C := -195$$

I am using the subscript C to indicate that this is the celcius temperature (Mathcad does not do temperature conversions)

The formula to convert C to K is:

$$K = C + 273.15 \quad \text{From the back of your book}$$

$$C = K - 273.15 \quad \text{By rearranging the above equation}$$

The Appropriate equation for this problem is:

$$\text{LN2}_K := \text{LN2}_C + 273.15$$

$$\text{LN2}_K = 78.15 \quad \text{Temperature in kelvin}$$

The formula to convert C to F (From the back of your book) is:

$$F = \left(\frac{9}{5} \cdot C \right) + 32$$

Apply to this problem:

$$\text{LN2}_F := \left(\frac{9}{5} \cdot \text{LN2}_C \right) + 32$$

$$\text{LN2}_F = -319 \quad \text{Temperature in Fahrenheit}$$

ii. Body Temp, 98 F, in C and K

$$\text{temp}_F := 98$$

The Conversion factor from the back of your book:

$$C = \frac{5}{9} \cdot (F - 32)$$

$$\text{temp}_C := \frac{5}{9} \cdot (\text{temp}_F - 32)$$

$$\text{temp}_C = 36.66666667$$

iii. Paper Flashpoint, 450 F, in C and K

$$\begin{aligned} \text{LN2}_F &:= 450 \\ \text{LN2}_C &:= \left(\frac{5}{9}\right) \cdot (\text{LN2}_F - 32) & \text{LN2}_C &= 232.22222222 \\ \text{LN2}_K &:= \text{LN2}_C + 273.15 & \text{LN2}_K &= 505.37222222 \end{aligned}$$

iv. Derive Conversion for F to C based on freezing and boiling point of water.

$$\begin{aligned} \text{Freezing}_F &:= 32 & \text{Boiling}_F &:= 212 \\ \text{Freezing}_C &:= 0 & \text{Boiling}_C &:= 100 \end{aligned}$$

Each scale has a different number of degrees between freezing and boiling. This corresponds to different size "steps". Part of the conversion factor is to adjust the step scale.

$$\begin{aligned} F_{\text{steps}} &:= \text{Boiling}_F - \text{Freezing}_F & F_{\text{steps}} &= 180 \\ C_{\text{steps}} &:= \text{Boiling}_C - \text{Freezing}_C & C_{\text{steps}} &= 100 \end{aligned}$$

From this the conversion between step size (X_{F_C} for F to C and X_{C_F} for C to F) is:

$$F_{\text{steps}} = C_{\text{steps}} \cdot X_{F_C} \qquad C_{\text{steps}} = F_{\text{steps}} \cdot X_{C_F}$$

These rearrange to:

$$X_{F_C} := \frac{F_{\text{steps}}}{C_{\text{steps}}} \qquad X_{C_F} := \frac{C_{\text{steps}}}{F_{\text{steps}}}$$

$$X_{F_C} = 1.8 \qquad X_{C_F} = 0.55555556$$

$$X_{F_C} = 9 \frac{1}{5} \qquad X_{C_F} = 5 \frac{1}{9}$$

Finally we need to account for the different starting points for the two scales
 When converting F to C, first subtract 32, so both scales are at 0
 When converting C to F, add 32 when you are finished.

2. What is the volume of object that is 83.6 cm long, 10.62 cm wide, and 1.2 inches thick? What is the volume after 5.74 cm is cut off from the long edge?

$$\text{length} := 83.6 \cdot \text{cm} \quad \text{width} := 10.62 \cdot \text{cm} \quad \text{height} := 1.2 \cdot \text{in}$$

$$\text{volume} := \text{length} \cdot \text{width} \cdot \text{height}$$

After converting all three values to the same units

$$\text{volume} = 2.70611194 \times 10^3 \text{ cm}^3$$

How many significant figures should this number have?

If you cut 5.74 cm from the long edge, the new length is:

$$\text{length} := \text{length} - 5.74 \cdot \text{cm} \qquad \text{volume} := \text{length} \cdot \text{width} \cdot \text{height}$$

Which gives a new volume of: $\text{volume} = 2.52030951 \times 10^3 \text{ cm}^3$

How many significant digits should this number have?

3. Solve the following problems and express the results with the correct number of significant figures.

$$3.894 \cdot 2.16 = 8.41104 \quad 8.41$$

$$2.96 + 8.1 + 5.0214 = 16.0814 \quad 16.1$$

$$2.17 + 4.32 + 401.278 + 21.826 = 429.59400 \quad 429.59$$

$$\frac{485}{9.231} = 52.54035316 \quad 52.5$$

$$2.46 \cdot 2 = 4.92 \quad 5$$

$$9.146 - 9.137 = 9.00000000 \times 10^{-3} \quad 0.009$$

$$(5.12 \cdot 10^4) \cdot (6.8726 \cdot 10^{-9}) = 3.5187712 \times 10^{-4} \quad 3.52 \cdot 10^{-4}$$

4. The mass of the sun is 1.989×10^{30} kg and the radius is 696000 km. More on the sun. What is the average density of the

sun in the following units:

a. kg/km³ =

b. kg/m³ =

c. kg/cm³ =

d. g/cm³ =

$$\text{mass} := 1.989 \cdot 10^{30} \cdot \text{kg}$$

$$\text{mass} = 1.989 \times 10^{33} \text{ gm}$$

The equation for the volume of a sphere is:

$$r := 696000 \cdot \text{km} \quad \text{volume} := \frac{4}{3} \cdot \pi \cdot r^3$$

$$\text{volume} = 1.41226543 \times 10^{18} \text{ km}^3$$

Convert units:

to cubic meters:

$$\text{km}^3 = 1 \text{ km} \cdot \text{km} \cdot \text{km}$$

$$\text{km}^3 = 1 (10^3 \cdot \text{m}) \cdot (10^3 \cdot \text{m}) \cdot (10^3 \cdot \text{m})$$

$$\text{km}^3 = 1 \times 10^9 \text{ m}^3$$

$$1.41226543 \cdot 10^{18} \cdot \text{km}^3 \cdot \left(\frac{10^9 \cdot \text{m}^3}{\text{km}^3} \right) = 1.41226543 \times 10^{27} \text{ m}^3$$

$$\text{volume} = 1.41226543 \times 10^{27} \text{ m}^3$$

to cubic centimeters:

$$\text{m}^3 = 1 (10^2 \cdot \text{cm}) \cdot (10^2 \cdot \text{cm}) \cdot (10^2 \cdot \text{cm})$$

$$\text{m}^3 = 1 \times 10^6 \text{ cm}^3$$

$$1.41226543 \cdot 10^{27} \cdot \text{m}^3 \cdot \left(\frac{10^6 \cdot \text{cm}^3}{\text{m}^3} \right) = 1.41226543 \times 10^{33} \text{ cm}^3$$

$$\text{volume} = 1.41226543 \times 10^{33} \text{ cm}^3$$

So the density of the sun is:

$$\text{density} := \frac{\text{mass}}{\text{volume}}$$

$$\text{density} = 1.40837548 \times 10^{12} \text{ kg km}^{-3}$$

$$\text{density} = 1.40837548 \times 10^3 \text{ kg}\cdot\text{m}^{-3}$$

$$\text{density} = 1.40837548 \times 10^{-3} \text{ kg cm}^{-3}$$

$$\text{density} = 1.40837548 \text{ gm}\cdot\text{cm}^{-3}$$

How does this density relate to values you are familiar with? How many significant digits should the solution have?

5. In the opening scenes of the movie Raiders of the Lost Ark, Indiana Jones tries to remove a gold idol from a booby-trapped pedestal. He replaces the idol with a bag of sand of approximately equal volume. (Density of gold = 19.32 g/cm³; density of sand 3 g/cm³.) {from Zumdahl, Chemistry, Saunders (1994).

a. Did he have a reasonable chance of not activating the mass-sensitive booby trap?

NO, the gold is 7 times more dense than the sand. The Idol would be much heavier.

b. in a later scene he and an unscrupulous guide play catch with the idol. Assume that the volume of the idol is about 1.0 L. If it were solid gold, what mass would the idol have? Is playing catch with it plausible?

$$\text{volume} := 1 \cdot \text{liter}$$

$$\text{volume} = 1 \times 10^3 \text{ cm}^3$$

$$\text{density} := 19.32 \cdot \frac{\text{gm}}{\text{cm}^3}$$

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{mass} := \text{density} \cdot \text{volume}$$

$$\text{mass} = 19.32 \text{ kg}$$

$$\text{mass} = 42.59330905 \text{ lb}$$

Can you toss a 40 pound bag of cement?

6. The world record for the hundred meter dash is 9.79 s (mens) and 10.49 s (womens).

- What is the average speed in m/s
- What is the average speed in km/h
- What is the average speed in ft/s
- What is the average speed in mi/h?
- At this speed, how long would it take to run 1.00×10^2 yards?

Men's Record

$$\text{distance} := 100 \cdot \text{m}$$

$$\text{time} := 9.79 \cdot \text{sec}$$

$$\text{speed} := \frac{\text{distance}}{\text{time}} \quad \text{speed} = 10.2145046 \text{ m} \cdot \text{s}^{-1}$$

$$\text{speed} = 36.77221655 \frac{\text{km}}{\text{hr}}$$

$$\text{speed} = 33.51215419 \frac{\text{ft}}{\text{sec}}$$

$$\text{speed} = 22.84919604 \frac{\text{mi}}{\text{hr}}$$

You can perform the unit conversions here. How many significant figures should the answer have? Be careful, how "precise" and accurate is the 100 in the "100 meter". Think about where the uncertainty is.

For 100 yards, convert to metric:

$$\text{distance} := 100 \cdot \text{yd} \quad \text{distance} = 91.44 \text{ m}$$

$$\text{time} := \frac{\text{distance}}{\text{speed}}$$

$$\text{time} = 8.951976 \text{ s}$$

Women's Record

$$\text{distance} := 100 \cdot \text{m}$$

$$\text{time} := 10.49 \cdot \text{sec}$$

$$\text{speed} := \frac{\text{distance}}{\text{time}} \quad \text{speed} = 9.53288847 \text{ m} \cdot \text{s}^{-1}$$

$$\text{speed} = 34.31839847 \frac{\text{km}}{\text{hr}}$$

$$\text{speed} = 31.27588079 \frac{\text{ft}}{\text{sec}}$$

$$\text{speed} = 21.32446418 \frac{\text{mi}}{\text{hr}}$$

For 100 yards, convert to metric:

$$\text{distance} := 100 \cdot \text{yd} \quad \text{distance} = 91.44 \text{ m}$$

$$\text{time} := \frac{\text{distance}}{\text{speed}}$$

$$\text{time} = 9.592056 \text{ s}$$

7. The mercury concentration in a polluted lake is 0.0125 $\mu\text{g Hg/L}$, what is the total mass in kilograms of mercury in the lake if the lake is approximately circular with a diameter of 10 miles and an average depth of 50 ft?

First, find the surface area of the lake:

$$\text{diameter} := 10 \cdot \text{mi}$$

$$\text{mi} = 1.609344 \times 10^3 \text{ m}$$

$$\text{diameter} = 10 \text{ mi} \cdot \left(\frac{1.609344 \cdot 10^3 \cdot \text{m}}{\text{mi}} \right)$$

$$\text{diameter} = 1.609344 \times 10^4 \text{ m}$$

$$r := \frac{1}{2} \cdot \text{diameter}$$

$$r = 8.04672 \times 10^3 \text{ m}$$

$$\text{area} := \pi \cdot r^2$$

$$\text{area} = 2.03417191 \times 10^8 \text{ m}^2$$

From the surface area and the depth, find the volume

$$\text{depth} := 50 \cdot \text{ft}$$

$$\text{depth} = 50 \text{ ft} \cdot \left(\frac{12 \cdot \text{in}}{1 \cdot \text{ft}} \right) \cdot \left(\frac{2.54 \cdot \text{cm}}{1 \cdot \text{in}} \right) \cdot \left(\frac{1 \cdot \text{m}}{100 \cdot \text{cm}} \right)$$

$$\text{depth} = 15.24 \text{ m}$$

$$\text{volume} := \text{area} \cdot \text{depth}$$

$$\text{volume} = 3.10007798 \times 10^9 \text{ m}^3$$

Next convert the concentration to cubic meters:

$$\mu\text{g} := 10^{-6} \cdot \text{gm}$$

$$\text{concentration}_{\text{Hg}} := 0.0125 \cdot \frac{\mu\text{g}}{\text{liter}}$$

$$\text{concentration}_{\text{Hg}} = 0.0125 \left(\frac{\mu\text{g}}{\text{liter}} \right) \cdot \left(\frac{10^{-6} \cdot \text{gm}}{\mu\text{g}} \right) \cdot \left(\frac{1 \cdot \text{liter}}{1000 \cdot \text{mL}} \right) \cdot \left(\frac{1 \cdot \text{mL}}{1 \cdot \text{cm}^3} \right) \cdot \left(\frac{100 \cdot \text{cm}}{\text{m}} \right)^3$$

$$\text{concentration}_{\text{Hg}} = 1.250 \times 10^{-5} \text{ gm} \cdot \text{m}^{-3}$$

The concentration is the mass of mercury in each cubic meter of water. To find the total mass:

$$\text{mass}_{\text{Hg}} := \text{concentration}_{\text{Hg}} \cdot \text{volume}$$

$$\text{mass}_{\text{Hg}} = 3.87509748 \times 10^4 \text{ gm}$$

$$\text{mass}_{\text{Hg}} = 38.75097479 \text{ kg}$$

$$\text{mass}_{\text{Hg}} = 0.043 \text{ ton}$$

This is a rather large amount of mercury. How many significant figures should this answer have?