

Chem 145, Problem Set, Chapter 1.

Marathon Problem:

Adapted from Zumdahl *Chemistry*, 1993. Page 39

A cylindrical bar of gold that is 1.5 in high and 0.25 in in diameter has a mass of 23.1984 g, as determined on an analytical balance. An empty graduated cylinder is weighed on a triple beam balance and has a mass of 73.47 g. After pouring a small amount of liquid into the graduated cylinder, the mass is 79.16 g. When the gold cylinder is placed in the graduated cylinder (the liquid covers the top of the gold cylinder), the volume indicated on the graduated cylinder is 8.5 mL. Assume that the temperatures of the gold bar and the liquid are 86°F.

What is the density of the gold at 30°C?

What is the density of the liquid at 86°F?

What is the density of the liquid at 300 K?

The first part of answering this question is to determine what information is required, and decide how you want to solve the question. Start with finding the density of the gold at 30 C. What information do you need?

Recall: $\text{density} = \frac{\text{mass}}{\text{volume}}$

Since you are looking for the density, you need the mass and the volume to solve the problem.

The mass is easy, that is given in the problem:

$$\text{mass}_{\text{gold}} := 23.1984 \cdot \text{gm} \quad (\text{NOTE: gm stands for gram in Mathcad.})$$

So next we need the volume. But, first what is the point of the temperature? Is mass constant with temperature? Is volume constant with temperature? Think about some examples in everyday life. Why does a Hot Air balloon work? Why do they place expansion joints in the highway?

Yes, volume changes with temperature. So, first what temperature is the gold bar in the problem?

A cylindrical bar of gold that is 1.5 in high and 0.25 in in diameter.... Assume that the temperatures of the gold bar and the liquid are 86°F.

So, is 86 °F the same as 30 °C?

$$\text{Temp}_F := 86$$

$$\text{Temp}_C := (\text{Temp}_F - 32) \cdot \frac{5}{9}$$

$$\text{Temp}_C = 30$$

So, the gold bar is at 30 °C and everything is set to find the volume of the gold cylinder.

Volume = length · area general formula for the volume of an object

area = $\pi \cdot r^2$ The area of a circle, recall this from geometry.

Convert diameter and length to cm:

$$1 \cdot \text{in} = 2.54 \cdot \text{cm}$$

Conversion factor from your book.

$$\text{diameter} := 0.25 \cdot \text{in} \cdot \left(\frac{2.54 \cdot \text{cm}}{1 \cdot \text{in}} \right)$$

diameter = 0.635 · cm Remember how the inches will cancel, to give the distance in cm?

$$\text{length} := 1.5 \cdot \text{in} \cdot \left(\frac{2.54 \cdot \text{cm}}{1 \cdot \text{in}} \right)$$

length = 3.81 · cm

Calculate the area of the circle:

$$r := \frac{1}{2} \cdot \text{diameter}$$

$$\text{area} := \pi \cdot r^2$$

$$\text{area} = 0.317 \cdot \text{cm}^2$$

NOTE: This is a MathCad document. The bold =, is just an equation. The := defines a variable. The regular = displays the result of the calculation.

Calculate the volume of the gold cylinder:

$$\text{volume}_{\text{gold}} := \text{area} \cdot \text{length}$$

$$\text{volume}_{\text{gold}} = 1.207 \cdot \text{cm}^3$$

$$\text{volume}_{\text{gold}} = 1.207 \cdot \text{mL} \quad \text{Since } 1 \text{ mL} = 1 \text{ cm}^3$$

Now calculate the density of the gold:

$$\text{density}_{\text{gold}} := \frac{\text{mass}_{\text{gold}}}{\text{volume}_{\text{gold}}}$$

$$\text{density}_{\text{gold}} = 19.226 \cdot \frac{\text{gm}}{\text{mL}}$$

The density of the gold at 30 C.

Finally, double check the answer. Is this a reasonable value for the density of gold? Where could you find the density of gold? How many significant figures should the answer have? (NOTE: the mathematics software I use for this does not do anything to track significant figures.)

Next we can calculate the density of the liquid at 86 F. (Do we need to worry about the temperature this time? Since we are calculating the density, we need the mass of the liquid and the volume of the liquid. This information is given in the problem. But it is presented in a roundabout way.

For the mass:

An empty graduated cylinder is weighed on a triple beam balance and has a mass of 73.47 g. After pouring a small amount of liquid into the graduated cylinder, the mass is 79.16 g.

So the mass of the liquid is the difference between the mass of the cylinder (the first weight), and the mass of the cylinder + the liquid (the second weight). This is written as a mathematical statement in this form:

$$\begin{aligned} \text{mass}_{\text{cylinder}} &:= 73.47 \cdot \text{gm} \\ \text{mass}_{\text{cylinder}} + \text{mass}_{\text{liquid}} &= 79.16 \cdot \text{gm} \end{aligned}$$

By moving the mass of the cylinder to the right side of the equation (by subtracting from both sides) this rearranges to:

$$\begin{aligned} \text{mass}_{\text{liquid}} &:= 79.16 \cdot \text{gm} - \text{mass}_{\text{cylinder}} \\ \text{mass}_{\text{liquid}} &= 5.69 \cdot \text{gm} \end{aligned}$$

For the volume:

When the gold cylinder is placed in the graduated cylinder (the liquid covers the top of the gold cylinder), the volume indicated on the graduated cylinder is 8.5 mL.

This means we know the volume of the liquid + the gold cylinder. Since we found the volume of the gold cylinder above, this can be solved as follows:

$$\begin{aligned} \text{volume}_{\text{gold}} &= 1.207 \cdot \text{mL} \\ \text{volume}_{\text{gold}} + \text{volume}_{\text{liquid}} &= 8.5 \cdot \text{mL} \end{aligned}$$

This rearranges, by moving the volume of the gold to the right.

$$\begin{aligned} \text{volume}_{\text{liquid}} &:= 8.5 \cdot \text{mL} - \text{volume}_{\text{gold}} \\ \text{volume}_{\text{liquid}} &= 7.293 \cdot \text{mL} \end{aligned}$$

Now we can solve for the density of the liquid at 86 F:

$$\begin{aligned} \text{density}_{\text{liquid}} &:= \frac{\text{mass}_{\text{liquid}}}{\text{volume}_{\text{liquid}}} \\ \text{density}_{\text{liquid}} &= 0.78 \cdot \frac{\text{gm}}{\text{mL}} \end{aligned}$$

Can we solve for the density of the liquid at 300 K? (Hint, do you know what temperature this is?)