Solutions to Chapter 5, Limiting Reagent Problem Set

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1.The Kingston Steam Plant burns 14,000 tons of coal each day and generates 1010 kilowatts-hours of electricity each year (enough for 700,000 homes). Coal is primarily carbon which undergoes combustion in the following reaction:

$$C(s) + O_2\,(g) \rightarrow CO_2(g)$$

a.How many grams of oxygen is required for the combustion of 1 day's coal?
b.How much carbon dioxide is produced each day?

Coal is the limiting reageint in this reaction (*unless the earth runs out of oxygen, but then we are all in really big trouble*). So first calculate the number of moles of coal.

$$\text{Mass}_{\text{coal}} := 14000 \cdot \text{ton}$$

$$\text{Mass}_{\text{coal}} = 1.4 \cdot 10^{4} \cdot \text{ton} \cdot \left(\frac{2000 \cdot \text{lb}}{1 \cdot \text{ton}}\right) \cdot \left(\frac{453.6 \cdot \text{gm}}{\text{lb}}\right)$$

$$\text{Mass}_{\text{coal}} = 1.27 \cdot 10^{10} \cdot \text{gm}$$

$$\text{MW}_{\text{C}} := 12.011 \cdot \text{gm} \cdot \text{mole}^{-1}$$

$$\text{Mole}_{\text{C}} := \frac{\text{Mass}_{\text{coal}}}{\text{MW}_{\text{C}}} \qquad \text{Mole}_{\text{C}} = 1.057 \cdot 10^{9} \cdot \text{mole}$$

From the balanced reaction; for every mole of carbon, one mole of oxygen is required, and one mole of CO_2 is produced.

Mass of O_2

$$\text{Mole}_{\text{O2}} := \text{Mole}_{\text{C}}$$

$$\text{MW}_{\text{O2}} := 2 \cdot 15.9994 \cdot \text{gm} \cdot \text{mole}^{-1}$$

$$\text{Mass}_{\text{O2}} := \text{Mole}_{\text{O2}} \cdot \text{MW}_{\text{O2}} \qquad \text{Mass}_{\text{O2}} = 3.384 \cdot 10^{10} \cdot \text{gm}$$

Mass of CO_2

$$\text{Mole}_{\text{CO2}} := \text{Mole}_{\text{C}}$$

$$\text{MW}_{\text{CO2}} := (12.011 + 2 \cdot 15.9994) \cdot \text{gm} \cdot \text{mole}^{-1}$$

$$\text{Mass}_{\text{CO2}} := \text{Mole}_{\text{CO2}} \cdot \text{MW}_{\text{CO2}} \qquad \text{Mass}_{\text{CO2}} = 4.654 \cdot 10^{10} \cdot \text{gm}$$

2. CO_2 is removed from the atomsphere by trees and converted into cellulose. The basic reaction for this is:

$$6\ CO_2\ (g) + 5\ H_2O\ (l) \rightarrow C_6H_{10}O_5\ (s) + 6\ O_2\ (g)$$

a. How many grams of water are required to process the CO_2 from one day's electrical production at the Kingston Steam Plant?
b. How much oxygen is produced by this process?
c. How much cellulose is produced?
d. If you assume that a tree weigh's 2 tons, how many trees are required to process the CO_2 produced in one day?

From the balanced chemical reaction, five moles of water are required for 6 moles of carbon dioxide:

$$Mole_{H2O} := Mole_{CO2} \cdot \frac{5}{6}$$

$$Mole_{H2O} = 8.812 \cdot 10^{8} \cdot mole$$

$$MW_{H2O} := (2 \cdot 1.00794 + 15.9994) \cdot gm \cdot mole^{-1}$$

$$Mass_{H2O} := Mole_{H2O} \cdot MW_{H2O} \qquad Mass_{H2O} = 1.587 \cdot 10^{10} \cdot gm$$

$$Density_{H2O} := 1 \cdot \frac{gm}{mL}$$

$$Volume_{H2O} := \frac{Mass_{H2O}}{Density_{H2O}} \qquad Volume_{H2O} = 1.587 \cdot 10^{10} \cdot mL$$

$$Volume_{H2O} = 4.194 \cdot 10^{6} \cdot gal$$

From the balanced chemical reaction, 6 moles of O_2 are produced from 6 moles of carbon dioxide:

$$Mole_{O2} := Mole_{CO2} \cdot \frac{6}{6}$$

$$Mole_{O2} = 1.057 \cdot 10^{9} \cdot mole$$

$$MW_{O2} := (2 \cdot 15.9994) \cdot gm \cdot mole^{-1}$$

$$Mass_{O2} := Mole_{O2} \cdot MW_{O2}$$

$$Mass_{O2} = 3.384 \cdot 10^{10} \cdot gm$$

Notice that this is identical to the mass of O_2 used in the combustion reaction. All the oxygen is recovered by the tree.

From the balanced chemical reaction, 1 mole of cellulose is produced from 6 moles of carbon dioxide:

$$Mole_{cellulose} := Mole_{CO2} \cdot \frac{1}{6}$$

$$Mole_{cellulose} = 1.762 \cdot 10^{8} \cdot mole$$

$$MW_{cellulose} := (12.011 \cdot 6 + 1.00794 \cdot 10 + 15.9994 \cdot 5) \cdot gm \cdot mole^{-1}$$

$$Mass_{cellulose} := Mole_{cellulose} \cdot MW_{cellulose}$$

$$Mass_{cellulose} = 2.858 \cdot 10^{10} \cdot gm$$

Assuming that a tree weighs 2 tons and is all cellulose:

$$tree := 2 \cdot ton$$

$$tree = 1.814 \cdot 10^{6} \cdot gm$$

$$Mass_{cellulose} = 2.857 \cdot 10^{7} \cdot kg \left(\frac{1 \cdot tree}{1.814 \cdot 10^{6} \cdot gm} \right)$$

$$Mass_{cellulose} = 15749 \cdot tree$$

So from this "back of the envelope" calculation, about 16 thousand trees must be grown to absorb the carbon dioxide produced in one day by this power plant.

3.A typical automobile gets 30 miles per gallon of gas and drives 12,000 miles every year. Assuming that octane (C_8H_{18}, density 0.7025 g cm-1) is a principal component of gasoline;

a.How much oxygen is required for a car to run for 1 year?
b.How much CO_2 is produced by the car in 1 year?
c.How many trees are required to remove the CO_2 produced by the car in 1 year?

Let's start with a balanced reaction:

$$2\ C_8H_{18} + 25\ O_2 \rightarrow 16\ CO_2 + 18\ H_2O$$

Next calculate the amout of octane used:

$$mileage := 30 \cdot mi \cdot gal^{-1}$$

$$distance := 12000 \cdot mi$$

$$volume_{gas} := \frac{distance}{mileage}$$

$$volume_{gas} = 4 \cdot 10^{2} \cdot gal$$

$$volume_{gas} = 1.514 \cdot 10^{6} \cdot mL$$

$density_{gas} := 0.7025 \cdot gm \cdot cm^{-3}$

$Mass_{gas} := density_{gas} \cdot volume_{gas}$ $\qquad Mass_{gas} = 1.064 \cdot 10^{6} \cdot gm$

$MW_{octane} := (12.011 \cdot 8 + 1.00794 \cdot 18) \cdot gm \cdot mole^{-1}$

$$Mole_{octane} := \frac{Mass_{gas}}{MW_{octane}}$$

$Mole_{octane} = 9.312 \cdot 10^{3}$

From the balanced chemical reaction, 25 moles of O_2 are required for 2 moles of octane:

$$Mole_{O2} := Mole_{octane} \cdot \frac{25}{2}$$

$Mole_{O2} = 1.164 \cdot 10^{5} \cdot mole$

$MW_{O2} := (2 \cdot 15.9994) \cdot gm \cdot mole^{-1}$

$Mass_{O2} := Mole_{O2} \cdot MW_{O2}$ $\qquad Mass_{O2} = 3.725 \cdot 10^{6} \cdot gm$

$Mass_{O2} = 4.106 \cdot ton$

From the balanced chemical reaction, 16 moles of CO_2 are produced by 2 moles of octane:

$$Mole_{CO2} := Mole_{octane} \cdot \frac{16}{2}$$

$Mass_{CO2} := Mole_{CO2} \cdot MW_{CO2}$ $\qquad Mass_{CO2} = 3.278 \cdot 10^{6} \cdot gm$

$Mass_{CO2} = 3.614 \cdot ton$

From the balanced chemical reaction, 1 mole of cellulose is produced from 6 moles of carbon dioxide:

$Mole_{cellulose} := Mole_{CO2} \cdot \frac{1}{6}$ $\qquad Mole_{cellulose} = 1.242 \cdot 10^{4} \cdot mole$

$Mass_{cellulose} := Mole_{cellulose} \cdot MW_{cellulose}$ $\qquad Mass_{cellulose} = 2.013 \cdot 10^{6} \cdot gm$

$Mass_{cellulose} = 1.1 \cdot tree$

4.Modern instrumental techniques are capable of detecting lead in a milliliter sample at picomolar concentration.

a.How many moles of lead are in the sample?

b.What is the mass of lead in this sample?

c.How many grams of sodium chloride would be required to precipitate all the lead in this sample as lead (II) chloride?

d.What would the mass of the lead (II) chloride precipitate be?

Calculate the amount of lead in the sample from the concentration and the total volume

$\text{Concentration}_{Pb} := 10^{-12} \cdot \text{mole} \cdot \text{liter}^{-1}$

$\text{Volume}_{Pb} := 1 \cdot \text{mL}$ $\text{Volume}_{Pb} = 1 \cdot 10^{-3} \cdot \text{liter}$

$\text{Mole}_{Pb} := \text{Concentration}_{Pb} \cdot \text{Volume}_{Pb}$ $\text{Mole}_{Pb} = 1 \cdot 10^{-15} \cdot \text{mole}$

$\text{MW}_{Pb} := 207.2 \cdot \text{gm} \cdot \text{mole}^{-1}$

$\text{Mass}_{Pb} := \text{MW}_{Pb} \cdot \text{Mole}_{Pb}$

$\text{Mass}_{Pb} = 2.072 \cdot 10^{-13} \cdot \text{gm}$

First write out the balanced equation:

Pb^{2+} + 2 NaCl --> $PbCl_2$ + 2 Na^{1+} (*Note: the lead is an ion in aqueous solution*)

From the balanced reaction, 2 moles of NaCl are required for each mole of Pb^{2+}

$\text{Mole}_{NaCl} := \text{Mole}_{Pb} \cdot \frac{2}{1}$ $\text{Mole}_{NaCl} = 2 \cdot 10^{-15} \cdot \text{mole}$

$\text{MW}_{NaCl} := (22.989768 + 35.4527) \cdot \text{gm} \cdot \text{mole}^{-1}$ $\text{MW}_{NaCl} = 58.442 \cdot \text{gm} \cdot \text{mole}^{-1}$

$\text{Mass}_{NaCl} := \text{MW}_{NaCl} \cdot \text{Mole}_{NaCl}$ $\text{Mass}_{NaCl} = 1.169 \cdot 10^{-13} \cdot \text{gm}$

From the balanced reaction, 1 mole of $PbCl_2$ is produced for each mole of Pb^{2+}

$\text{Mole}_{PbCl2} := \text{Mole}_{Pb} \cdot \frac{1}{1}$ $\text{Mole}_{PbCl2} = 1 \cdot 10^{-15} \cdot \text{mole}$

$\text{MW}_{PbCl2} := (107.2 + 2 \cdot 35.4527) \cdot \text{gm} \cdot \text{mole}^{-1}$ $\text{MW}_{PbCl2} = 1.781 \cdot 10^{2} \cdot \text{gm} \cdot \text{mole}^{-1}$

$\text{Mass}_{NaCl} := \text{MW}_{NaCl} \cdot \text{Mole}_{NaCl}$ $\text{Mass}_{NaCl} = 1.169 \cdot 10^{-13} \cdot \text{gm}$