

Chemistry 145 Lecture Problems

Heat Capacity and Specific Heat

Heating Water for Chocolate:

250 mL of water

4.184 J g⁻¹ K⁻¹

From 15 C to 90 C

Determine the mass of the water:

$$\text{Volume}_{\text{H}_2\text{O}} := 250 \cdot \text{mL}$$

$$\text{Density}_{\text{H}_2\text{O}} := 1.00 \cdot \text{gm} \cdot \text{mL}^{-1}$$

$$\text{Mass}_{\text{H}_2\text{O}} := \text{Volume}_{\text{H}_2\text{O}} \cdot \text{Density}_{\text{H}_2\text{O}}$$

$$\text{Mass}_{\text{H}_2\text{O}} = 250 \text{ gm}$$

Determine the temperature change:

$$T_{\text{initial}} := (273.15 + 15) \cdot \text{K}$$

$$T_{\text{final}} := (273.15 + 90) \cdot \text{K}$$

NOTE: Mathcad only works in absolute temperature units. But since Centigrade and kelvin degrees are the same "size", either unit will work for this.

$$\Delta T := T_{\text{final}} - T_{\text{initial}}$$

$$\Delta T = 75 \text{ K}$$

Calculate the energy required for this temperature change:

$$C_{\text{H}_2\text{O}} := 4.184 \cdot \text{joule} \cdot \text{gm}^{-1} \cdot \text{K}^{-1}$$

$$\text{Energy} := \text{Mass}_{\text{H}_2\text{O}} \cdot C_{\text{H}_2\text{O}} \cdot \Delta T$$

$$\text{Energy} = 7.845 \times 10^4 \text{ joule}$$

How much will the same amount of energy change the temperature of a 250 gram piece of gold?

$$C_{\text{Au}} := 0.128 \cdot \text{joule} \cdot \text{gm}^{-1} \cdot \text{K}^{-1}$$

$$\text{Mass}_{\text{gold}} := 250 \cdot \text{gm}$$

$$\Delta T := \frac{\text{Energy}}{\text{Mass}_{\text{gold}} \cdot C_{\text{Au}}}$$

$$\Delta T = 2452 \text{ K}$$

Heat a 50 g piece of copper to 500 C. Then place in 1 liter of water at 20 C. What is the final temperature?

This is one of the first examples this semester of solving two equations simultaneously. Lets start by looking at what information we have and what we can do with it.

For copper

$$\text{Mass}_{\text{Cu}} := 50 \cdot \text{gm}$$

$$T_{\text{Cu}} := (273.15 + 500) \cdot \text{K}$$

For water

$$\text{Volume}_{\text{H}_2\text{O}} := 1 \cdot \text{liter}$$

$$T_{\text{H}_2\text{O}} := (273.15 + 20) \cdot \text{K}$$

The problem only provides amount and temperature. But if you look in your book, you can find the specific heat capacity for each of these materials. Since specific heat capacity is given for a gram of material, we need to convert the amount of water from a volume to a mass. So we need the density of water.

$$C_{\text{Cu}} := 0.385 \cdot \text{joule} \cdot \text{gm}^{-1} \cdot \text{K}^{-1}$$

$$C_{\text{H}_2\text{O}} = 4.184 \cdot \text{joule} \cdot \text{gm}^{-1} \cdot \text{K}^{-1}$$

$$d_{\text{H}_2\text{O}} := 0.99821 \cdot \text{gm} \cdot \text{mL}^{-1}$$

From *CRC Handbook of Chemistry and Physics, 73rd ed.*

$$\text{Mass}_{\text{H}_2\text{O}} := \text{Volume}_{\text{H}_2\text{O}} \cdot d_{\text{H}_2\text{O}}$$

$$\text{Mass}_{\text{H}_2\text{O}} = 998.21 \text{ gm}$$

When the copper is placed in the water, the temperature of the copper will decrease and the temperature of the water will increase. The copper will loose energy and the water will gain energy. The equation for the energy gained (or lost) based upon temperature change, mass, and heat capacity is:

$$E = \text{mass} \cdot C \cdot \Delta T$$

Where

$$\Delta T = (T_{\text{final}} - T_{\text{initial}})$$

Combines to:

$$E = \text{mass} \cdot C \cdot (T_{\text{final}} - T_{\text{initial}})$$

Now we can write two equations. One for the heat lost by the copper, and a second for the heat gained by the water. These equations are:

$$E_{\text{Cu}} = \text{Mass}_{\text{Cu}} \cdot C_{\text{Cu}} \cdot (T_{\text{final_H2O}} - T_{\text{Cu}}) \quad \text{Equation \#1}$$

$$E_{\text{H2O}} = \text{Mass}_{\text{H2O}} \cdot C_{\text{H2O}} \cdot (T_{\text{final_H2O}} - T_{\text{H2O}}) \quad \text{Equation \#2}$$

For these two equations, we do not know the Energy released by the copper, the energy gained by the water, the final temperature of the copper, or the final temperature of the water. We currently have four unknowns and only two equations. But we need at least one equation for each unknown to find a solution.

But recall that the energy lost by the copper must be equal to the energy gained by the water. This statement is written mathematically as:

$$E_{\text{H2O}} = -E_{\text{Cu}} \quad \text{Equation \#3}$$

NOTE: The negative sign is because the energy leaving the copper is going to the water.

Also, since the final temperature of the water and the final temperature of the copper are the same (that is the whole point of the experiment). We can write another equation:

$$T_{\text{final_H2O}} = T_{\text{final_Cu}} \quad \text{Equation \#4}$$

Now we are up to four equations with four unknowns. This may now be solved. We'll just do some substitution to show this:

$$T_{\text{final}} = T_{\text{final_H2O}}$$

$$T_{\text{final}} = T_{\text{final_Cu}}$$

So we rewrite the original equations as:

$$E_{\text{Cu}} = \text{Mass}_{\text{Cu}} \cdot C_{\text{Cu}} \cdot (T_{\text{final}} - T_{\text{Cu}})$$

$$E_{\text{H2O}} = \text{Mass}_{\text{H2O}} \cdot C_{\text{H2O}} \cdot (T_{\text{final}} - T_{\text{H2O}})$$

This "used up" equation #4, but now we are down to three unknowns. Next from equation #3.

$$\text{Mass}_{\text{Cu}} \cdot C_{\text{Cu}} \cdot (T_{\text{final}} - T_{\text{Cu}}) = -\text{Mass}_{\text{H2O}} \cdot C_{\text{H2O}} \cdot (T_{\text{final}} - T_{\text{H2O}})$$

This step "used" equation #4, but now we are down to 1 unknown and one equation. Which we can now solve. You can substitute in the known values, I am going to let mathcad rearrange the above equation and then solve.

$$\text{Mass}_{\text{Cu}} \cdot C_{\text{Cu}} \cdot (T_{\text{final}} - T_{\text{Cu}}) = -\text{Mass}_{\text{H}_2\text{O}} \cdot C_{\text{H}_2\text{O}} \cdot (T_{\text{final}} - T_{\text{H}_2\text{O}})$$

$$T_{\text{final}} := \frac{-(-\text{Mass}_{\text{Cu}} \cdot C_{\text{Cu}} \cdot T_{\text{Cu}} - \text{Mass}_{\text{H}_2\text{O}} \cdot C_{\text{H}_2\text{O}} \cdot T_{\text{H}_2\text{O}})}{\text{Mass}_{\text{Cu}} \cdot C_{\text{Cu}} + \text{Mass}_{\text{H}_2\text{O}} \cdot C_{\text{H}_2\text{O}}}$$

$$T_{\text{final}} = 295.35222286 \text{ K}$$

Another approach to solving this problem is as follows:

Heat a 10 g piece of copper to 500 C. Then place in 1 liter of water at 20 C. What is the final temperature?

Instead of solving the problem by solving all the equations at the same time, we can use a numerical problem solving technique called successive approximation. To do this, you set up the problem, then guess the answer. You check this guess to see if it is high or low, and based upon the result you make another guess. This means you have to repeat the calculation several times, but the algebra is easier. This is a very useful technique for solving problems with very complex equations.

What we know:

For copper

$$\text{Mass}_{\text{Cu}} = 50 \text{ gm}$$

$$T_{\text{Cu}} = 773.15 \text{ K}$$

$$C_{\text{Cu}} = 0.385 \text{ joule} \cdot \text{gm}^{-1} \cdot \text{K}^{-1}$$

For water

$$\text{Mass}_{\text{H}_2\text{O}} = 998.21 \text{ gm}$$

$$T_{\text{H}_2\text{O}} = 293.15 \text{ K}$$

$$C_{\text{H}_2\text{O}} = 4.184 \text{ joule} \cdot \text{gm}^{-1} \cdot \text{K}^{-1}$$

Guess the final temperature:

$$T_f := 500 \cdot \text{K}$$

$$\text{Energy of Cu} \quad \text{Mass}_{\text{Cu}} \cdot C_{\text{Cu}} \cdot (T_f - T_{\text{Cu}}) = -5.2581375 \times 10^3 \text{ joule}$$

$$\text{Energy of Water} \quad \text{Mass}_{\text{H}_2\text{O}} \cdot C_{\text{H}_2\text{O}} \cdot (T_f - T_{\text{H}_2\text{O}}) = 8.63911226 \times 10^5 \text{ joule}$$

This calculation shows that the energy lost by the copper is not great enough to warm the water to this final temperature. The final temperature must be lower. So make a new guess:

$$T_f := 400 \cdot \text{K}$$

$$\text{Energy of Cu} \quad \text{Mass}_{\text{Cu}} \cdot C_{\text{Cu}} \cdot (T_f - T_{\text{Cu}}) = -7.1831375 \times 10^3 \text{ joule}$$

$$\text{Energy of Water} \quad \text{Mass}_{\text{H}_2\text{O}} \cdot C_{\text{H}_2\text{O}} \cdot (T_f - T_{\text{H}_2\text{O}}) = 4.46260162 \times 10^5 \text{ joule}$$

This calculation shows that the energy lost by the copper is still not great enough to warm the water to this final temperature. The final temperature must be lower. So make a new guess:

$$T_f := 300 \cdot \text{K}$$

$$\text{Energy of Cu} \quad \text{Mass}_{\text{Cu}} \cdot C_{\text{Cu}} \cdot (T_f - T_{\text{Cu}}) = -9.1081375 \times 10^3 \text{ joule}$$

$$\text{Energy of Water} \quad \text{Mass}_{\text{H}_2\text{O}} \cdot C_{\text{H}_2\text{O}} \cdot (T_f - T_{\text{H}_2\text{O}}) = 2.86090979 \times 10^4 \text{ joule}$$

This calculation shows that the energy lost by the copper is still not great enough to warm the water to this final temperature. The final temperature must be lower. So make a new guess:

$$T_f := 200 \cdot \text{K}$$

$$\text{Energy of Cu} \quad \text{Mass}_{\text{Cu}} \cdot C_{\text{Cu}} \cdot (T_f - T_{\text{Cu}}) = -1.10331375 \times 10^4 \text{ joule}$$

$$\text{Energy of Water} \quad \text{Mass}_{\text{H}_2\text{O}} \cdot C_{\text{H}_2\text{O}} \cdot (T_f - T_{\text{H}_2\text{O}}) = -3.89041966 \times 10^5 \text{ joule}$$

This calculation shows that both the copper and the water loose energy. That makes sense since the water started at 293.15 K. The final temperature must be higher. This was not a very good guess. So make a new guess:

$$T_f := 295 \cdot \text{K}$$

$$\text{Energy of Cu} \quad \text{Mass}_{\text{Cu}} \cdot C_{\text{Cu}} \cdot (T_f - T_{\text{Cu}}) = -9.2043875 \times 10^3 \text{ joule}$$

$$\text{Energy of Water} \quad \text{Mass}_{\text{H}_2\text{O}} \cdot C_{\text{H}_2\text{O}} \cdot (T_f - T_{\text{H}_2\text{O}}) = 7.72654468 \times 10^3 \text{ joule}$$

This calculation shows that the energy lost by the copper is greater than the amount required to warm the water to this final temperature. The final temperature must be higher So make a new guess:

$$T_f := 297.5 \cdot \text{K}$$

$$\text{Energy of Cu} \quad \text{Mass}_{\text{Cu}} \cdot C_{\text{Cu}} \cdot (T_f - T_{\text{Cu}}) = -9.1562625 \times 10^3 \text{ joule}$$

$$\text{Energy of Water} \quad \text{Mass}_{\text{H}_2\text{O}} \cdot C_{\text{H}_2\text{O}} \cdot (T_f - T_{\text{H}_2\text{O}}) = 1.81678213 \times 10^4 \text{ joule}$$

This calculation shows that the energy lost by the copper is still not great enough to warm the water to this final temperature. The final temperature must be lower. So make a new guess:

$$T_f := 296 \cdot \text{K}$$

$$\text{Energy of Cu} \quad \text{Mass}_{\text{Cu}} \cdot C_{\text{Cu}} \cdot (T_f - T_{\text{Cu}}) = -9.1851375 \times 10^3 \text{ joule}$$

$$\text{Energy of Water} \quad \text{Mass}_{\text{H}_2\text{O}} \cdot C_{\text{H}_2\text{O}} \cdot (T_f - T_{\text{H}_2\text{O}}) = 1.19030553 \times 10^4 \text{ joule}$$

This calculation shows that the energy lost by the copper is still not great enough to warm the water to this final temperature. The final temperature must be lower. So make a new guess:

$$T_f := 295.5 \cdot K$$

$$\text{Energy of Cu} \quad \text{Mass}_{\text{Cu}} \cdot C_{\text{Cu}} \cdot (T_f - T_{\text{Cu}}) = -9.1947625 \times 10^3 \text{ joule}$$

$$\text{Energy of Water} \quad \text{Mass}_{\text{H}_2\text{O}} \cdot C_{\text{H}_2\text{O}} \cdot (T_f - T_{\text{H}_2\text{O}}) = 9.8148 \times 10^3 \text{ joule}$$

This calculation shows that the energy lost by the copper is still not great enough to warm the water to this final temperature. But now we are getting very close. The final temperature must be lower. So make a new guess:

$$T_f := 295.25 \cdot K$$

$$\text{Energy of Cu} \quad \text{Mass}_{\text{Cu}} \cdot C_{\text{Cu}} \cdot (T_f - T_{\text{Cu}}) = -9.199575 \times 10^3 \text{ joule}$$

$$\text{Energy of Water} \quad \text{Mass}_{\text{H}_2\text{O}} \cdot C_{\text{H}_2\text{O}} \cdot (T_f - T_{\text{H}_2\text{O}}) = 8.77067234 \times 10^3 \text{ joule}$$

This calculation shows that the energy lost by the copper is greater than the amount required to warm the water to this final temperature. The final temperature must be higher. So make a new guess:

$$T_f := 295.35 \cdot K$$

$$\text{Energy of Cu} \quad \text{Mass}_{\text{Cu}} \cdot C_{\text{Cu}} \cdot (T_f - T_{\text{Cu}}) = -9.19765 \times 10^3 \text{ joule}$$

$$\text{Energy of Water} \quad \text{Mass}_{\text{H}_2\text{O}} \cdot C_{\text{H}_2\text{O}} \cdot (T_f - T_{\text{H}_2\text{O}}) = 9.18832341 \times 10^3 \text{ joule}$$

This is the same answer that was found using the exact expression. If you program the equations into Excel, it is very easy to change the final temperature to get the correct answer.

