# Chemistry 146 Lecture Problems
# Generic Buffer

Let's take a look at how this buffer system is useful.

Given a weak acid system where:

$HA + H_2O \leftrightarrow A^- + H_3O^+$

And the equlibrium starting conditions for 1 liter of solution:

$$C_{HA} := 1 \cdot \frac{mole}{liter} \quad C_A := 1 \cdot \frac{mole}{liter} \quad C_{H3O} := 1 \cdot \frac{mole}{liter} \qquad M := \frac{mole}{liter}$$

This gives an equlibrium constant ($K_a$):

$$K_a := \frac{C_A \cdot C_{H3O}}{C_{HA}}$$

$$K_a = 1 \circ M$$

Now, lets disrupt the equlibrium by adding 0.5 moles NaOH

$OH^-$ is a strong base so it will react completely with any best proton donor.

$H_3O^+ + OH^- \leftrightarrow 2\ H_2O$ where $K_w = 10^{-14}$

Almost all of the $OH^-$ is converted to $H_2O$.

As a result of this reaction the New, non-equlibrium concentrations are approximately:

$$C_{H3O} := C_{H3O} - 0.5 \cdot M \qquad C_{HA} = 1 \circ M \qquad C_A = 1 \circ M$$

$$C_{H3O} = 0.5 \circ M \qquad C_{H3O} = 0.5 \circ M$$

Assuming X moles shift, the new equlibrium may be described as:

$$C_{H3O} = C_{H3O} + X \qquad C_A = C_A + X \qquad C_{HA} = C_{HA} - X$$

Then $K_a$ is:

$$K_a = \frac{(C_{H3O} + X) \cdot (C_A + X)}{(C_{HA} - X)}$$

Substituting in for the known values results in the expression:

$$1=\frac{(0.5+X)\cdot(1.0+X)}{(1.0-X)}$$

expands to

$$1=\frac{\left(.5+1.5\cdot X+X^2\right)}{(1.0-X)}$$

rearanges to

$$-.5+2.5\cdot X+X^2=0$$

has solution(s)

$$X=\begin{bmatrix}-2.686140661634507165\\ .186140661634507165\end{bmatrix}$$

Picking the physically meaningful root gives:

$$X:=0.18614\cdot M$$

Which gives new equlibrium concentrations as:

$$C_{H3O}:=C_{H3O}+X \quad C_A:=C_A+X \quad C_{HA}:=C_{HA}-X$$

$$C_{H3O}=0.68614\circ M \quad C_A=1.18614\circ M \quad C_{HA}=0.81386\circ M$$

Even though 0.5 mole of NaOH was added, the [H3O+], changed by a much smaller amount. If you assume the volume of the solution was constant at 1 L:

$$pH_{initial}:=-\log(1) \qquad pH_{initial}=0$$

$$pH_{final}:=-\log(0.686) \qquad pH_{final}=0.16368$$

Compare this to the pH change if you add 0.5 mole NaOH to 1 L of water

$$pH_{water}:=-\log\left(10^{-7}\right) \qquad pH_{water}=7$$

$$pOH_{base}:=-\log(0.5) \qquad pOH_{base}=0.30103$$

$$pH_{base}:=14-pOH_{base} \qquad pH_{base}=13.69897$$