Solutions to: Nuclear Chemistry Problem Set

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1.Calculate the Energy produced by the following fusion reaction:

2H + 3H --> 4He + 1n

Given the following exact mass measurements:

$$MW_{2H} := 2.0140 \cdot gm \cdot mole^{-1}$$

$$MW_{3H} := 3.01605 \cdot gm \cdot mole^{-1}$$

$$MW_{4He} := 4.0026031 \cdot gm \cdot mole^{-1}$$

$$MW_{n} := 1.008665012 \cdot gm \cdot mole^{-1}$$

Start by calculating the mass of the products and of the reactants:

$$Mass_{product} := MW_{4He} + MW_{n}$$

$$Mass_{product} = 5.011268112 \circ gm \cdot mole^{-1}$$

$$Mass_{reactant} := MW_{2H} + MW_{3H}$$

$$Mass_{reactant} = 5.03005 \circ gm \cdot mole^{-1}$$

Notice that the mass of the products is less than the mass of the reactants. What happened to this mass? It is converted into energy. So now find the change in mass:

$$\Delta Mass := Mass_{product} - Mass_{reactant}$$

$$\Delta Mass = -0.018781888 \circ gm \cdot mole^{-1}$$

Given the change in mass, we can use the following equation to find the energy released (since mass is lost)

$$E = m \cdot c^2$$

$$c := 299792458 \cdot \frac{m}{sec}$$

$$E_{rxn} := \Delta Mass \cdot c^2$$

$$E_{rxn} = -1.6880319106 \cdot 10^{12} \frac{kg \cdot m^2}{s^2 \cdot mol}$$

$$E_{rxn} = -1.6880319106 \cdot 10^{12} \circ joule \cdot mole^{-1}$$

2.Based upon the results above, calculate how much deuterium and tritium would be required to run a 100 MW power plant for 1 year.

$$MW := 10^6 \cdot watt$$

$$Power := 100 \cdot MW$$

$$watt = 1 \circ \frac{joule}{sec}$$

$$time := 1 \cdot yr$$

$$time = 3.1556926 \cdot 10^7 \ s$$

$$Energy := Power \cdot time$$

$$Energy = 3.1556926 \cdot 10^{15} \ J$$

Since the reaction produces

$$E_{rxn} = -1.6880319106 \cdot 10^{12} \circ joule \cdot mole^{-1}$$

$$Mole := \frac{Energy}{E_{rxn}}$$

$Mole = -1.8694507966 \cdot 10^3 \ mol$ This is the number of moles of "reaction" required to produce the energy needed

$Mass_{2H} := -MW_{2H} \cdot Mole$ $Mass_{2H} = 3.7650739043 \ kg$

$Mass_{3H} := -MW_{3H} \cdot Mole$ $Mass_{3H} = 5.638357075 \ kg$

3.Calculate the binding energy for an alpha particle given the following information:

$MW_{proton} := 1.007276470 \cdot gm \cdot mole^{-1}$

$MW_{neutron} := 1.008665012 \cdot gm \cdot mole^{-1}$

$MW_{\alpha} := 4.0015061 \cdot gm \cdot mole^{-1}$

Given that an alpha particle contains two protons and two neutrons, it's mass would be:

$MW_{\alpha\_parts} := 2 \cdot MW_{proton} + 2 \cdot MW_{neutron}$

$MW_{\alpha\_parts} = 4.031882964 \circ gm \cdot mole^{-1}$

Notice that this is different than the actual mass of the alpha particle. The mass defect is this difference.

$Mass\_defect := MW_{\alpha\_parts} - MW_{\alpha}$

$Mass\_defect = 0.030376864 \circ gm \cdot mole^{-1}$

This mass defect accounts for the binding energy of the nucleus, this mass is the source of the energy that holds the nucleus together.

$E_{binding} := Mass\_defect \cdot c^2$

$E_{binding} = 2.7301363834 \cdot 10^{12} \circ joule \cdot mole^{-1}$

4.Based upon C-14 dating, with a current count of 7 min-1, a half life of 5730 years, and an initial count of 13.6 min-1, what is the age of the sample? What would be the counts after 10,000 years?

The first order integrated rate equation relates amounts at various times for a first order process. Since radioactive decay is a first order process:

$$\ln\left(\frac{N_o}{N_t}\right) = k \cdot t$$

We are given the initial and final "count" for the sample. The count simply refers to how much radiation is present. Since the radiation comes from the C-14, the amount of radiation (or the number of counts) is directly related to the amount of C-14. This is our N. It does not matter what the units are, since they cancel. The important part is that the units cancel. They could be mole, gram, molarity, pounds, liters, or counts.

$$N_o := 13.6 \cdot \text{min}^{-1}$$

$$N_t := 7 \cdot \text{min}^{-1}$$

We are also given the half life for the radioactive decay of C-14. The half life is related to the rate constant. Derived from the integrated rate equation, the half life is the amount of time it takes for half of the reactant to react. i.e.: Nt = 1/2 No. Substituting this into the equation we get:

$$t_{half} := 5730 \cdot \text{yr}$$

$$\ln\left(\frac{N_o}{\frac{1}{2} \cdot N_o}\right) = k \cdot t_{half}$$

$$\ln(2) = k \cdot t_{half}$$

$$k := \frac{\ln(2)}{t_{half}}$$

$$k = 1.2096809434 \cdot 10^{-4} \circ \text{yr}^{-1}$$

Now we can put these values into the integrated rate equation and solve for t.

$$t := \ln\left(\frac{N_o}{N_t}\right) \cdot \frac{1}{k}$$

$$t = 5.4903703933 \cdot 10^{3} \circ \text{yr}$$

The next part of the question asks what the count would be after 10,000 years. This requires solving the integrated rate equation for Nt

$t := 10000 \cdot yr$

$$\ln\left(\frac{N_o}{N_t}\right) = k \cdot t$$

$$\ln\left(\frac{N_t}{N_o}\right) = -k \cdot t$$

$$\frac{N_t}{N_o} = e^{-k \cdot t}$$

$N_t := N_o \cdot e^{-k \cdot t}$

$N_t = 4.0567771354 \circ min^{-1}$

This answer is about right since 10,000 years is a bit less than two half lives, so the radiation level should be a bit more than 1/4 the initial.

5.Based upon U-238 dating, with a half life of 4.5*109 years, a sample of rock contains 1.502 grams of U-238 and 0.312 grams of Pb, what is the age of the rock. What assumptions are made?

Starting with the half life to find the rate constant. As above:

$t_{half} := 4.5 \cdot 10^9 \cdot yr$

$$k := \frac{\ln(2)}{t_{half}}$$

$k = 1.5403270679 \cdot 10^{-10} \circ yr^{-1}$

Next, the initial and final amount of U-235:

$$U238_{final} := 1.502 \cdot gm$$

$$Pb206_{final} := 0.312 \cdot gm$$

The Pb-206 comes from the U-238 that was initially present. So we need to add the amount of lead to the amount of uranium. However, we can not simply add the mass, since many other products were produced along the reaction pathway. We can, however, add the moles (like any chemistry reaction, moles produced equal moles reacted)

$$Mole_{Pb} := \frac{Pb206_{final}}{\left(206 \cdot gm \cdot mole^{-1}\right)}$$

$$Mole_{Pb} = 1.5145631068 \cdot 10^{-3}\ mol$$

$$Mole_{U\_reacted} := Mole_{Pb}$$

$$Mass_{U\_reacted} := Mole_{U\_reacted} \cdot 238 \cdot gm \cdot mole^{-1}$$

$$Mass_{U\_reacted} = 0.3604660194 \circ gm$$

$$U238_{initial} := U238_{final} + Mass_{U\_reacted}$$

$$U238_{initial} = 1.8624660194 \circ gm$$

Now we may solve the integrated rate equation to solve for the age of the sample

$$t := \ln\left(\frac{U238_{initial}}{U238_{final}}\right) \cdot \frac{1}{k}$$

$$t = 1.3964818098 \cdot 10^{9} \circ yr$$

Solving this problem requires making the assumption that this reaction is first order, that the rate constant has not changed, that there was no lead initially present in the sample, and that none of the lead or uranium were lost from the sample.

6.Based upon K-40 dating, with a half life of 1.3*109 years and a current analysis of potassium (1.502 grams) and argon (0.312 grams), what is the age of this rock?

This problem is essentially identical to the one above, so we get:

$$t_{half} := 1.3 \cdot 10^{9} \cdot yr$$

$$k := \ln(2) \cdot \frac{1}{t_{half}}$$

$$k = 5.3319013889 \cdot 10^{-10} \circ yr^{-1}$$

The final amount of K40:

$$K_{final} := 1.502 \cdot gm$$

The amount of K that reacted:

$$Ar := 0.312 \cdot gm$$

$$Mole_{Ar} := \frac{Ar}{39.948 \cdot gm \cdot mole^{-1}}$$

$$Mole_{Ar} = 7.8101531992 \cdot 10^{-3}\ mol$$

$$Mole_{K\_reacted} := Mole_{Ar}$$

$$K_{initial} := K_{final} + Mole_{K\_reacted} \cdot \left(39.0983 \cdot gm \cdot mole^{-1}\right)$$

$$K_{initial} = 1.8073637128 \circ gm$$

And solving for time with the integrated rate equation:

$$t := \ln\left(\frac{K_{initial}}{K_{final}}\right) \cdot \frac{1}{k}$$

$$t = 3.4710266459 \cdot 10^{8} \circ yr$$

Solving this problem requires making the assumption that this reaction is first order, that the rate constant has not changed, that there was no argon initially present in the sample, and that none of the argon or potassium were lost from the sample.

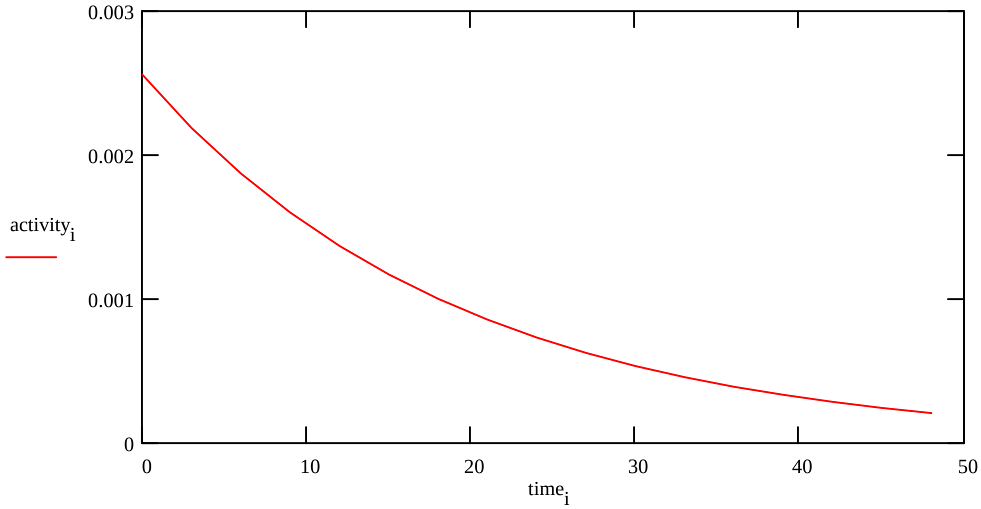
7.Radioactive 123I was produced in a nuclear reactor. The a series of experiments was performed to determine the concentration of 123I at different times. The results are given in the table below.

$i := 0, 1 .. 16$

$time_i :=$ $activity_i :=$

$time_i$	$activity_i$
0	0.00256
3	0.002189
6	0.001873
9	0.001602
12	0.00137
15	0.001171
18	0.001002
21	0.000857
24	0.000733
27	0.000627
30	0.000536
33	0.000458
36	0.000392
39	0.000335
42	0.000287
45	0.000245
48	0.00021

a.Graph this data



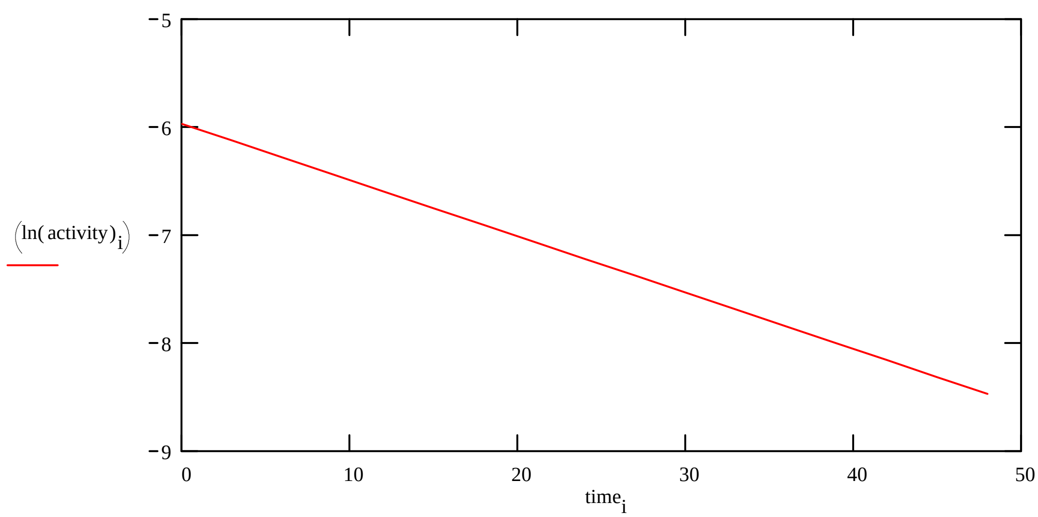
$$\text{slope}(\text{time}, \text{activity}) = -4.5320261438 \cdot 10^{-5}$$

$$\text{intercept}(\text{time}, \text{activity}) = 2.0551568627 \cdot 10^{-3}$$

Note: Since this data is not a straight line, the slope and intercept are not really meaningful.

b.Graph this data for a first order kinetics plot.

For first order kinetics, a ln vs. time plot will be linear.



c. What is the rate constant for the radioactive decay of 123I?

The rate constant for a first order reaction is determined from the slope of the ln vs. t graph.

$\mathrm{slope}(\mathrm{time}, \ln(\mathrm{activity})) = -0.0521236005$

$\mathrm{intercept}(\mathrm{time}, \ln(\mathrm{activity})) = -5.9676633241$

$k := -\mathrm{slope}(\mathrm{time}, \ln(\mathrm{activity})) \cdot \mathrm{hr}^{-1}$

$k = 0.0521236005 \, \mathrm{hr}^{-1}$

d. Based upon this data what is the half life for 123I?

$t_{half} := \ln(2) \cdot \frac{1}{k}$

$t_{half} = 13.2981446725 \, \mathrm{hr}$

e. How much 123I will remain after 3 days?

$t := 3 \cdot \mathrm{day}$

$t = 72 \, \mathrm{hr}$

From the integrated rate equation:

$$\frac{N_t}{N_o} = e^{-k \cdot t}$$

$e^{-k \cdot t} = 0.0234496611$

$e^{-k \cdot t} = 2.3449661072 \, \%$

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