Properties of a Gaussian Distribution

This worksheet generates a gaussian distribution for a given average and standard deviation.

Average: \( \mu := 0 \)
Standard deviation: \( \sigma := 1 \)
Amplitude: \( A := 1 \)

The equation for a gaussian distribution (with an amplitude of \( A \)):

\[
y(x) := A \cdot e^{-\frac{1}{2} \left( \frac{x-\mu}{\sigma} \right)^2}
\]

Range to Plot:
start := -5
stop := 5
\[ x := \text{start} + \left( \frac{\text{stop} - \text{start}}{100} \right) \]

Random Normal Distribution
Equation for a gaussian distribution with an area of 1:

\[ y_{\text{area}}(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \]
Finding the area under part of a gaussian distribution:

\[
\text{start} := -1 \\
\text{stop} := 0 \\
x_a := \text{start} + \left\lfloor \frac{\text{stop} - \text{start}}{100} \right\rfloor \cdot \text{stop}
\]

Random Normal Distribution (area = 1)

Numerical integration of a gaussian curve (taken using 100 steps):

\[
\sum_{x_a} \frac{y_{\text{area}}(x)}{100} = 0.345
\]

Integration of a gaussian curve using calculus:

\[
\int_{\text{start}}^{\text{stop}} \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \, dx = 0.3413447461
\]

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