

Week 7 Problem Set - Acid Base Buffers

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$$M := \frac{\text{mole}}{\text{liter}}$$

1. For a 100.0 mL solution that is 0.1 M acetic acid calculate the concentration of acetate ion, hydronium ion, and the pH.

$$K_a := 1.8 \cdot 10^{-5} \cdot M$$

$$K_w := 1.0 \cdot 10^{-14} \cdot M^2$$

The concentration of the acid is

$$C_{\text{acid}} := 0.1 \cdot M$$

$$C_{\text{acid}} = 0.1 \text{ M}$$

The equilibrium solution for	HA	+	H ₂ O	<->	H ₃ O ⁺	+	A ⁻
Initial Concentraion	C_{acid}						
Equilibrium Concentration	$C_{\text{acid}} - X$				X		X

Equilibrium expression:

$$K_a = \frac{X^2}{C_{\text{acid}} - X}$$

Assuming X is smaller than [acid], this reduces to:

$$K_a = \frac{X^2}{C_{\text{acid}}}$$

Solving for X:

$$X := \sqrt{K_a \cdot C_{\text{acid}}}$$

$$X = 1.3416 \times 10^{-3} \text{ M}$$

Equilibrium Concentrations:

$$C_{\text{HA}} := C_{\text{acid}} - X$$

$$C_{\text{HA}} = 0.099 \text{ M}$$

$$C_{\text{H}_3\text{O}} := X$$

$$C_{\text{H}_3\text{O}} = 1.342 \times 10^{-3} \text{ M}$$

$$C_{\text{A}} := X$$

$$C_{\text{A}} = 1.342 \times 10^{-3} \text{ M}$$

$$\text{pH} := -\log(C_{\text{H}_3\text{O}} \cdot M^{-1})$$

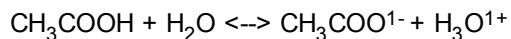
$$\text{pH} = 2.872$$

$$\text{pOH} := 14 - \text{pH}$$

$$\text{pOH} = 11.128$$

2. For a 100.0 mL solution that is 0.1 M acetic acid and 0.1 M in sodium acetate calculate the concentration of acetate ion, hydronium ion, and the pH.

Acetic acid is a weak acid so it is in an equilibrium:



Sodium acetate is a salt, so when placed in water it will dissociate. This reaction goes to completion:



Since the dissociation of the salt, goes to completion, start by dealing with that reaction.

	NaCH ₃ COO	-->	Na ¹⁺	+	CH ₃ COO ¹⁻
Initial	0.1·M		0·M		0·M
Change	-0.1·M		■ + 0.1·M		■ + 0.1·M
Final	0·M		0.1·M		0.1·M

Now, look at the equilibrium expression for acetic acid.

	CH ₃ COOH	+	H ₂ O	<-->	CH ₃ COO ¹⁻	+	H ₃ O ¹⁺
Initial	0.1·M				0.1·M		0·M
Change	-X				■ + X		■ + X
Final	0.1 - X				0.1 + X		X

To solve for X, we need to look at the equilibrium expression:

$$K_a = \frac{C_{\text{CH}_3\text{COO}^-} \cdot C_{\text{H}_3\text{O}^+}}{C_{\text{CH}_3\text{COOH}}}$$

$$1.8 \cdot 10^{-5} = \frac{(0.1 + X) \cdot (X)}{(0.1 - X)}$$

$$X = \left(\begin{array}{l} 1.7993523496892227 \cdot 10^{-5} \\ -0.10003599352349689223 \end{array} \right)$$

Simplifying the expression:

$$1.8 \cdot 10^{-5} = \frac{(0.1 + X) \cdot (X)}{(0.1 - X)}$$

$$1.8 \cdot 10^{-5} = \frac{(0.1) \cdot (X)}{(0.1)}$$

$$\underset{\text{www}}{X} := 1.8 \cdot 10^{-5} \quad \text{Essentially the same as the exact solution.}$$

Which gives final concentrations for everything as:

$$\text{CH}_3\text{COOH} \quad 0.1 - X = 0.1$$

$$\text{CH}_3\text{COO}^{1-} \quad 0.1 + X = 0.1$$

$$\text{H}_3\text{O}^{1+} \quad X = 1.8 \times 10^{-5} \quad \text{Notice this value, it is not a coincidence.}$$

$$\underset{\text{www}}{\text{pH}} := -\log(X) \quad \text{pH} = 4.745$$

Finally, look at K_w (Since this is the smallest equilibrium constant):

	$2 \text{H}_2\text{O}$	\rightleftharpoons	H_3O^{1+}	+	OH^{1-}
Initial			$1.8 \cdot 10^{-5}$	0	
Change			$\blacksquare + X$		$\blacksquare + X$
Final			$1.8 \cdot 10^{-5} + X$		X

Solve for X using the equilibrium expression:

$$K_w = C_{\text{H}_3\text{O}^+} \cdot C_{\text{OH}^-}$$

$$1.0 \cdot 10^{-14} = (1.8 \cdot 10^{-5} + X) \cdot (X)$$

$$X = \left(\begin{array}{c} -1.8000555538409837511 \cdot 10^{-5} \\ 5.555384098375108 \cdot 10^{-10} \end{array} \right)$$

Or simplify the expression:

$$1.0 \cdot 10^{-14} = (1.8 \cdot 10^{-5}) \cdot (X)$$

$$\underset{\text{www}}{X} := 5.55556 \cdot 10^{-10}$$

Which is identical to the exact solution (the meaningful root)

Which gives the final equilibrium concentrations as:

$$\text{OH}^{1-} \quad X = 5.556 \times 10^{-10}$$

3. For a 100.0 mL solution that is 0.1 M acetic acid and 0.05 M in sodium acetate calculate the concentration of acetate ion, hydronium ion, and the pH.

Since the dissociation of the salt, goes to completion, start by dealing with that reaction.

	NaCH ₃ COO	-->	Na ¹⁺	+	CH ₃ COO ¹⁻
Initial	0.05·M		0·M		0·M
Change	-0.05·M		■ + 0.05·M		■ + 0.05·M
Final	0·M		0.05·M		0.05·M

Now, look at the equilibrium expression for acetic acid.

	CH ₃ COOH	+	H ₂ O	<-->	CH ₃ COO ¹⁻	+	H ₃ O ¹⁺
Initial	0.1·M				0.05·M		0·M
Change	-X				■ + X		■ + X
Final	0.1 - X				0.05 + X		X

To solve for X, we need to look at the equilibrium expression:

$$K_a = \frac{C_{\text{CH}_3\text{COO}^-} \cdot C_{\text{H}_3\text{O}^+}}{C_{\text{CH}_3\text{COOH}}} \quad 1.8 \cdot 10^{-5} = \frac{(0.05 + X) \cdot (X)}{(0.1 - X)} \quad X = \left(\begin{array}{l} 0.000035961189828184678851 \\ -0.050053961189828184679 \end{array} \right)$$

Simplifying the expression:

$$1.8 \cdot 10^{-5} = \frac{(0.05 + X) \cdot (X)}{(0.1 - X)}$$

$$1.8 \cdot 10^{-5} = \frac{(0.05) \cdot (X)}{(0.1)}$$

$$X := 0.000036$$

Essentially the same as the exact solution.

Which gives final concentrations for everything as:

CH ₃ COOH	0.1 - X = 0.1
CH ₃ COO ¹⁻	0.05 + X = 0.05
H ₃ O ¹⁺	X = 3.6 × 10 ⁻⁵

$$\text{pH} := -\log(X) \quad \text{pH} = 4.444$$

Finally, look at K_w (Since this is the smallest equilibrium constant): Or simplify the expression:

$$1.0 \cdot 10^{-14} = (3.6 \cdot 10^{-5}) \cdot (X)$$

$$X := 2.78 \cdot 10^{-10}$$

Which gives the final equilibrium concentrations as:

OH ¹⁻	X = 2.78 × 10 ⁻¹⁰
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4. For a 100.0 mL solution that is 0.1 M acetic acid and 0.15 M in sodium acetate calculate the concentration of acetate ion, hydronium ion, and the pH.

Since the dissociation of the salt, goes to completion, start by dealing with that reaction.

	NaCH_3COO	\rightarrow	Na^{1+}	+	$\text{CH}_3\text{COO}^{1-}$
Initial	0.15·M		0·M		0·M
Change	-0.15·M		■ + 0.15·M		■ + 0.15·M
Final	0·M		0.15·M		0.15·M

Now, look at the equilibrium expression for acetic acid.

	CH_3COOH	+	H_2O	\leftrightarrow	$\text{CH}_3\text{COO}^{1-}$	+	H_3O^{1+}
Initial	0.1·M				0.15·M		0·M
Change	-X				■ + X		■ + X
Final	0.1 - X				0.15 + X		X

To solve for X, we need to look at the equilibrium expression:

$$K_a = \frac{C_{\text{CH}_3\text{COO}^-} \cdot C_{\text{H}_3\text{O}^+}}{C_{\text{CH}_3\text{COOH}}} \quad 1.8 \cdot 10^{-5} = \frac{(0.15 + X) \cdot (X)}{(0.1 - X)} \quad X = \left(\begin{array}{l} 0.000011997600671773524907 \\ -0.15002999760067177352 \end{array} \right)$$

Simplifying the expression:

$$1.8 \cdot 10^{-5} = \frac{(0.15 + X) \cdot (X)}{(0.1 - X)}$$

$$1.8 \cdot 10^{-5} = \frac{(0.15) \cdot (X)}{(0.1)}$$

$$X := 0.000012$$

Essentially the same as the exact solution.

Which gives final concentrations for everything as:

CH_3COOH	$0.1 - X = 0.1$
$\text{CH}_3\text{COO}^{1-}$	$0.15 + X = 0.15$
H_3O^{1+}	$X = 1.2 \times 10^{-5}$
$\text{pH} := -\log(X)$	$\text{pH} = 4.921$

Finally, look at K_w (Since this is the smallest equilibrium constant): Or simplify the expression:

$$1.0 \cdot 10^{-14} = (1.2 \cdot 10^{-5}) \cdot (X) \quad X := 8.33 \cdot 10^{-10}$$

Which gives the final equilibrium concentrations as:

$$\text{OH}^{1-} \quad X = 8.33 \times 10^{-10}$$

5. For a 100.0 mL solution that is 0.1 M acetic acid and 0.2 M in sodium acetate calculate the concentration of acetate ion, hydronium ion, and the pH.

Since the dissociation of the salt, goes to completion, start by dealing with that reaction.

	NaCH ₃ COO	→	Na ¹⁺	+	CH ₃ COO ¹⁻
Initial	0.15·M		0·M		0·M
Change	-0.15·M		■ + 0.2·M		■ + 0.2·M
Final	0·M		0.2·M		0.2·M

Now, look at the equilibrium expression for acetic acid.

	CH ₃ COOH	+	H ₂ O	↔	CH ₃ COO ¹⁻	+	H ₃ O ¹⁺
Initial	0.1·M				0.2·M		0·M
Change	-X				■ + X		■ + X
Final	0.1 - X				0.2 + X		X

To solve for X, we need to look at the equilibrium expression:

$$K_a = \frac{C_{\text{CH}_3\text{COO}^-} \cdot C_{\text{H}_3\text{O}^+}}{C_{\text{CH}_3\text{COOH}}} \quad 1.8 \cdot 10^{-5} = \frac{(0.2 + X) \cdot (X)}{(0.1 - X)} \quad X = \begin{pmatrix} 0.0000089987852186532639439 \\ -0.20002699878521865326 \end{pmatrix}$$

Simplifying the expression:

$$1.8 \cdot 10^{-5} = \frac{(0.2 + X) \cdot (X)}{(0.1 - X)}$$

$$1.8 \cdot 10^{-5} = \frac{(0.2) \cdot (X)}{(0.1)}$$

$$\underline{X} := 0.000009$$

Essentially the same as the exact solution.

Which gives final concentrations for everything as:

CH ₃ COOH	0.1 - X = 0.1
CH ₃ COO ¹⁻	0.15 + X = 0.15
H ₃ O ¹⁺	X = 9 × 10 ⁻⁶
$\underline{\text{pH}} := -\log(X)$	pH = 5.046

Finally, look at K_w (Since this is the smallest equilibrium constant): Or simplify the expression:

$$1.0 \cdot 10^{-14} = (1.2 \cdot 10^{-5}) \cdot (X) \quad \underline{X} := 8.33 \cdot 10^{-10}$$

Which gives the final equilibrium concentrations as:

OH ¹⁻	X = 8.33 × 10 ⁻¹⁰
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